DOCUMENT RESUME

ED 317 429 SE 051 348

TITLE	Developing Math Learning Skills: A Parallel Support Course for the Math-Anxious College Student.
INSTITUTION	New Mexico State Univ., Las Cruces.
SPONS AGENCY	Women's Educational Equity Act Program (ED), Washington, DC.
PUB DATE	89
NOTE	104p.
AVAILABLE FROM	WEEA Publishing Center, Educational Development Center, Inc., 55 Chapel Street, Newton, MA 02160 ($5.00).
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*College Mathematics; *Course Content; Course Descriptions; Courses; Females; Higher Education; Mathematics Achievement; *Mathematics Anxiety; Mathematics Curriculum; Mathematics Education; Mathematics Materials; *Mathematics Skills; *Minority Groups; *Remedial Mathematics; Remedial Programs; Test Anxiety

ABSTRACT

Anxiety toward mathematics is a persistent barrier to successful completion of a college degree for women and minorities. This program adapted materials and philosophies from other programs to meet the needs of students identified as impaired by mathematics anxiety in their pursuit of a college education. The program addresses both influencing students' attitudes and behavior and seeking to make changes in the standard learning environment. The background and objectives of the program are described. The 12 lessons included concern (1) Introductions and overview of math anxiety; (2) Collecting data and recording feelings toward math; (3) Examining math myths; (4) Developing a math vocabulary; (5) Reading mathematics; (6) Introducing problem solving; (7) Reducing test anxiety; (8) Evaluation; (9) Translating English into mathematics; (10) Discovering structure underlying algorithmic story problems; (11) Reviewing structure of algebra; and (12) Preparing for and taking examinations. In each lesson, the rationale, objectives, materials, activities, evaluation, and handouts are provided. The appendices contain materials on tutor information, student activities, and questionnaires. (YP)

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DEVELOPING MATH LEARNING SKILLS

A Parallel Support Course for the Math-Anxious Student

New Mexico State University
Las Cruces, NM

Women's Educational Equity Act Program
U.S. Department of Education

SE 051 348

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Developing Math Learning Skills

A Parallel Support Course for the Math-Anxious College Student

Developing Math Learning Skills

A Parallel Support Course for the Math-Anxious College Student

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The activity which is the subject of this report was produced under a grant from the U.S. Department of Education, under the auspices of the Women's Educational Equity Act. Opinions expressed herein do not necessarily reflect the position or policy of the Department, and no official endorsement should be inferred.

Printed and distributed by WEEA Publishing Center, 1989
Education Development Center, Inc., 55 Chapel Street
Newton, Massachusetts 02160

Cover design by Darcie Sanders

Code no.: 0702

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Preface

The Math Learning Project authors and staff have long believed that the teaching of mathematics must be available to all college students in a form in which the students experience the success of learning mathematics. We have intended to demystify the dread topic and to demonstrate how much of a sixth sense it actually is.

The success of this project is gratefully indebted to the following individuals, whose support and encouragement sustained us throughout:

Our devoted project assistants/tutors, Jacquie Kurland, Armando Torres, and John Miller

Dr. Karen Smith, Yvonne Flores, and Jessica Padilla from the Center for Learning Assistance

Director Kitty Berver and instructors from the Math Learning Center

Dr. Timothy Pettibone, Dr. Jack Cole, Norma Ruskey, Sherry Stewart, and Melissa Andersen from the College of Education

Our dedicated participants, *teachers* of the teachers, in all honesty

1

Background to the Math Learning Project

Introduction

Anxiety toward mathematics is a persistent barrier to successful completion of a college degree for women and minorities at New Mexico State University (NMSU), as it is at other universities and community colleges in the nation. NMSU requires all students to demonstrate proficiency in mathematics basic skills in addition to completing appropriate mathematics courses within a degree program. Negative feelings toward mathematics, as well as poor skills in mathematics and a lack of understanding of basic concepts, hinders or eliminates many women and minority students from participating in educational programs that lead to fulfilling and rewarding careers.

We felt that a comprehensive program, dealing with both the psychological and knowledge barriers associated with mathematics education, and providing instruction and counseling on both individual and group levels, was necessary for women and minority students to function at the educational levels required to meet the increasing demands of a technological society.

Women and Mathematics

According to the National Science Foundation (1982), wide differences persist in achievement and participation levels among students from different groups. Women have traditionally participated less than men in sciences, and members of minority groups have participated less and performed less well than whites on standardized science and mathematics achievement tests. Researchers cite a number of reasons for these differences, including unwillingness to study mathematics and science, social attitude, and perception of usefulness of the subject.

Unwillingness to Study Mathematics

Fennema (1976) concluded in a National Institute of Education report that "the problem with girls is not the ability to learn math but the willingness to study math" (87).

In support of this statement, Fox (1981) reported that "recent studies suggest that the small representation of women in quantitative fields may not be the result of innate differences in ability but may be caused by early decisions not to study mathematics" (9). Hackett (1985), in a path analysis, found evidence that mathematics anxiety influences choice of a mathematics-related college major. By deciding not to pursue mathematics in school, many women are also choosing not to study for careers in quantitative fields and are thereby limiting their career options and advancement opportunities.

The National Assessment of Educational Progress's fourth mathematics assessment in 1986 found a consistent advantage for males in the area of higher-level applications (Dossey et al. 1988). Campbell (1986) reported that girls take fewer math courses than boys. One more explanation for sex differences on test performance is that opportunities for learning and applying mathematics knowledge and skills outside of school may be greater for men.

Social Attitude

A great deal of attention has been paid to the influence of "significant others"—such as parents, teachers, counselors, and peers—on the continued study of mathematics. Students' attitudes toward the study of mathematics and their self-confidence, career interests, and values have been shown to influence course-taking decisions.

Although women make up about 44 percent of the labor force and hold 49 percent of the professional and related positions, they are markedly absent from the ranks of scientists and engineers. In 1986 women held about 28 percent of the positions in mathematics, physical science, and life science but only 4 percent of those in engineering (National Science Foundation 1988). While the employment of women in science and engineering fields has increased, women are still underrepresented. The situation, according to Fox (1981, 16), can be attributed to social attitudes that still inhibit the study of mathematics and discourage women from pursuing careers, rather than to basic sex differences in aptitude.

Perception of Usefulness

In the "Women and Mathematics National Survey" conducted in fall 1978, the perception of the usefulness of mathematics correlated with achievement test scores (Armstrong 1979). Furthermore, Tobias states: "Most people of both sexes stop taking mathematics before their education is complete. Girls who avoid mathematics and mathematics-related subjects may simply be getting the message sooner than boys that mathematics is unrewarding and irrelevant" (1978, 74). Campbell (1986) suggested that counselors and math teachers actively recruit students (particularly girls and minorities) by emphasizing the importance of math in career preparation and success.

Needs of Women and Minority Students

New Mexico, a center for advanced scientific work in energy, computer research, and space technology, is a state with a growth potential for high-technology industries. It is also the home of the oldest and most traditional cultures in the United States. Hispanics make up more than 40 percent of the state's population; American Indians, 9 percent. These two groups are far behind the nonminority population in terms of mainstream social, economic, and educational conditions. For example, 30 percent of all Hispanic adults in rural areas have dropped out of school prior to high school graduation and the dropout rate for American Indian adults is 50 percent (Sher 1978).

Few minority students are acquainted with successful professionals in mathematics-based careers, such as engineers, physicists, or mathematicians. Consequently, most are unaware of the career possibilities that proficiency in mathematics might open to them. In many cases, their families and teachers are similarly unaware of these possibilities and therefore do not encourage the students to concentrate on mathematics and science.

Many rural women in the Southwest face three difficulties in gaining appropriate educational and occupational opportunities: their rural residence, their gender, and their minority status (Amodeo et al. 1981). Rural women, like their urban counterparts, increasingly find themselves as heads of households working outside the home to survive. As a result, greater numbers of them continue to enter the labor force. They are largely confined to low-paying, low-status, unskilled jobs in clerical, service, or seasonal occupations (Chu 1980).

Growth and change in the Southwest are creating opportunities for women to break from the confines of occupational restrictions. Many new industries, especially in high-technology and energy-related fields, are locating in the region. Rural women in the Southwest need to learn how to benefit from and adapt to these changes.

An increasing number of mature women are seeking higher educational opportunities. More than 26 percent of the student population at NMSU are over twenty-five years old, and the trend for an older,

part-time, increasingly female student body is expected to continue. These returning students come to the campus with a variety of learning experiences. They are highly motivated students but bring many problems with them. Lack of confidence, unsupportive family members, lack of basic skills, and guilt and anxiety feelings frequently hinder their educational progress (Brown 1982).

Scientifically and technically trained tribal members are vitally important to natural resource development on reservations. Tribes are increasingly reluctant to hire non-Indians who do not share tribal values regarding land and natural resources. According to Lujan and Burr, "Until a corps of professional Indian people are trained for tribal leadership in engineering, science, business and agriculture, neither the Indian tribes nor the nation will derive all the possible benefits of resource development on Indian reservations" (1985, 596). Most American Indian students who indicate an interest in an engineering major do not finish in that major. In a 1984 study of three major universities in New Mexico, there were 58 percent fewer American Indian engineering students in the junior and senior classes than in the freshmen and sophomore classes.

According to Moore (1982), an area of difficulty for American Indian students appears to be hypothetical thought and abstraction based on hypotheses. The mathematics teacher must attempt to close the gap between the abstract and the real world of the American Indian student. States Moore: "The utilization of laboratory techniques is consistent with the Navajo philosophy of education. The Navajo verb for 'to teach' means 'to show'" (24).

Many American Indian students enter NMSU declaring a major in engineering, computer science, or some technical area. However, their ACT-Mathematics score and their performance on the NMSU Mathematics Placement Exam indicate they are ill prepared in mathematics, and so they begin to work their way through the mathematics curriculum required for their technical course of study at this basic skills level. For some, the remediation emphasis of the course does not identify their deficiencies or serve to overcome them, and so they are stymied in their chosen major before they actually begin.

Many of the students enrolled in the basic skills classes are women and minority students who express a great deal of anxiety toward mathematics. Often this anxiety prevents them from successfully completing the courses. Kogelman and Warren (1978) reported this condition as a panic in which normal functioning is impaired and the skills necessary for learning and performing become inaccessible. Comprehensive assistance for these students in dealing with math anxiety, developing mathematics study skills, and relearning mathematics concepts on a concrete basis will provide an adequate foundation for studying college-level mathematics. In addition, these students need to be taught reasoning and problem-solving skills.

Mathematics Program at New Mexico State University

The most common way for students at NMSU to meet the mathematics basic skills requirement is by successfully completing two non-degree-credit courses offered by the Department of Mathematics in the Mathematics Learning Center (MLC). Students are placed in these courses based on their results on the Mathematics Placement Exam and diagnostic exams; they begin at different levels of mastery in arithmetic and basic algebra. Instruction in these courses is provided by professional mathematics educators and carefully selected student tutors following a structured, individually paced program. Emphasis is on mastery of computational skills. More than three thousand students a semester enroll in the MLC program. Unfortunately, the size of the MLC program and restricted funding prohibit the staff from focusing on students with special mathematical learning difficulties. Since the MLC is directed at the underprepared math student, its population includes a large number of returning adult and/or minority women.

2

The Math Learning Project: Significance of the Program

The experience of many counselors for women and minorities in college, especially in technical majors, is that in some cases math anxiety presents a stumbling block so great that students abandon planned careers or end their university education in failure. Fox (1981) indicated a need for more experimental research to determine program characteristics that are most helpful to women. In our Math Learning Project we evaluated the effectiveness of a comprehensive, structured, semester-long program to parallel enrollment in the basic skills math classes.

This program adapted materials and philosophies from other more narrowly defined programs to meet the needs of those students identified as impaired by math anxiety in their pursuit of a college education. The innovative aspects of this program included attack on both the psychological and knowledge barriers associated with mathematics education, instruction in mathematics study skills, and provision for group study experiences to form the basis for an enduring study/support system to increase the likelihood that these students would complete course work leading to a degree.

Goals and Objectives

Intervention to remedy math anxiety can be aimed at influencing students' attitudes and behavior directly, at seeking to make changes in the standard learning environment, or at both. In this project we addressed both methods.

The goals of this one-year project were as follows:

1. to provide the opportunity for students, especially women and minorities, to examine the situations and attitudes that had created their anxiety toward mathematics
2. to work with students of similar background who had similar problems with mathematics
3. to help students relearn specific mathematics concepts that presented barriers to successful completion of higher mathematics courses in such a way that those concepts became a part of the students' cognitive understanding of mathematics
4. to create an academic environment in which women and minority students could successfully participate in college-level mathematics

These goals were accomplished through individual and group counseling, individual tutoring, and group instruction providing laboratory experiences.

The following were the objectives of this project:

1. to design instructional materials and counseling approaches for math-anxious women and minority students, following a careful analysis of these students' needs
2. to create a program consistent with the identified needs
3. to test the comprehensive program by conducting a one-semester course
4. to revise the program, write a final report, and prepare the program materials for publication

Institutional Needs Analysis

The project began in January 1986. To analyze the needs of women and minority students in mathematics education in general, we reviewed pertinent literature and investigated programs that attempted to meet these needs (Ortega and Reyes 1985; Roueche, Baker, and Roueche 1984; Threadgill-Sowder et al. 1983). To analyze the needs of this group on our campus, we conducted personal interviews with directors and professional staff members from the following offices:

Center for Learning Assistance
Mathematics Learning Center
Counseling and Student Development Center
Center for Improving Teaching and Learning of Mathematics
Adult Learning Center of the Dona Ana Branch Community College at NMSU
Special Student Services
Black Programs
Chicano Programs
American Indian Programs
Indian Resource Development Program
Returning Students' Association
Women's Center
Arts and Sciences Advising Center
Business Advising Center
Education Advising Center
College of Human and Community Services
Associate Dean of College of Engineering
Associate Dean of Agriculture and Home Economics

These interviews provided us with an overview of the services offered to students on the campus, insight into the perceived needs of the targeted group, and suggestions both for potential participants in the project and for tutors. The interviews also developed rapport across the campus and provided support for our project. A sample letter sent to these offices is in appendix A.

During the interviews we asked for observations regarding math learning, including the levels of mathematics with which students experienced the greatest difficulty, the reasons that students gave to explain the difficulty, the reasons that the interviewee felt the students experienced difficulty, and suggestions for alternate approaches to math learning. We also asked for information on training tutors and how they would identify a "successful" tutor.

The people whom we interviewed strongly supported our program and confirmed the need for it. These professionals encouraged us to design a program that would allow maximum participation from students and would help students learn how to ask questions. The interviewees suggested that we focus on reducing anxiety and building confidence by helping students to experience success. These professionals perceived that students needed assistance in translating mathematics vocabulary, developing math reading skills, and managing time. They felt that hands-on experiences and concrete examples would be particularly important. We were encouraged to provide training for our tutors in communication skills, learning-style differences, and above all, patience.

From these interviews, the area of greatest need for the population was identified as the transition from arithmetic to beginning algebra. The directors and professional staff identified students we could interview as potential participants. In addition, we advertised on campus for interested students who were experiencing difficulty with mathematics. (A sample application form is in appendix A.) Eighty-two students were referred by directors and professional staff, and thirty more students approached the project staff on their own; forty of these students were selected to participate in an hour-long interview.

Just prior to interviewing, two student tutor/research assistants were hired: a returning Anglo

female engineering major and a traditional-aged Hispanic male engineering major. (The job announcement listing the qualifications is in appendix B.) They were hired because of their mathematics background and their interest in and commitment to women and minority students. They were initially trained in communication and interviewing skills (Cohen 1978). A retired male math teacher volunteered to help, and he was also trained as a tutor. (A handout on interviewing techniques is in appendix B.)

Student Needs Analysis

Based on information from similar programs, data from support offices, and our own backgrounds in learning and mathematics, we designed an interview questionnaire for prospective students. This questionnaire involved both oral and written responses, to help meet the need of differing learning-style preferences. The purpose of the questionnaire was to help us understand the source of math anxiety, to identify problem-solving strategies that these students were currently using, and to receive input on how they would design an ideal math support class. (A copy of this questionnaire is in appendix C.)

From the students' interviews, we were able to reconfirm that math anxiety was a learned response to previous negative experiences. Most students could identify specific negative experiences that they had had in elementary and junior high school. It was also apparent that the students were limited in their approaches to studying mathematics. They conveyed to us that their ideal math class would be self-paced, with a focus on the understanding of basic concepts. They preferred a quiet setting with tutors and instructors readily available and lots of interaction among students and between instructors and students. They suggested having fewer than twenty students per class, and they wanted a mixture of ages and sexes. They wanted lots of visual aids and more detailed books. They described a good math teacher as one who is patient, caring, and concerned; who understands each student's view; who gives good explanations; who does not assume anything; who helps students; who uses different teaching methods; and who makes sure periodically that the student understands.

Math Learning Course

The forty interviews were completed at the end of the spring semester, and twenty students were selected to enroll in the Math Learning Project course for the fall semester.

During the summer of 1986, we compiled the results of our interviews, questionnaires, and research from successful programs. Based on these data, we began developing the program components: mathematics study skills, anxiety-coping skills, problem-solving strategies, individual and group counseling, laboratory experiences for relearning basic concepts, parallel support in related courses, and ways to devise a personal balance between life-style and math anxiety problems. One of the project staff members researched and developed a tutor training manual. We prepared our course outline, a syllabus, and instructional materials. Because the setting for doing math seemed particularly important to students, we located and decorated a room away from the mathematics building to serve as a combination office, classroom, and tutoring room.

A three-credit course graded satisfactory/unsatisfactory (S/U) was designed. Students attended a group class twice a week and scheduled a minimum of one hour per week of individual tutoring. In addition, they were required to be enrolled in an arithmetic or algebra math class at the university.

One hour per week of the Math Learning Project course was spent in reducing math anxiety, learning math study skills, developing math problem-solving skills, and so on. The other hour was spent in group tutoring and in relearning basic math concepts that related to students' math course. The individual tutoring provided assistance in understanding the material from the regular math class or, in some cases, in related science areas. An extensive file of supplementary worksheets was available to students.

Twenty-two students enrolled in the course. To meet scheduling needs, the group was divided

in half; one group met in the morning and the other in the afternoon. The tutors attended at least one section of the class each week.

The preformative evaluation collected from the interviews with professional staff members and from the students was used to design the structure and atmosphere for our course. We used our backgrounds in learning assistance and mathematics education, as well as our research into successful programs, to design the course content.

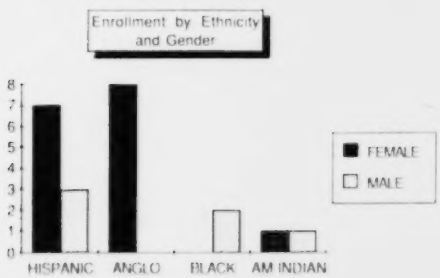
Individual tutoring logs were written for each student. These logs kept track of progress that each student made and areas in which students repeatedly needed help. The logs also provided another evaluation tool and different approaches that seemed to work well with particular students. (A copy of the tutoring log is in appendix B.)

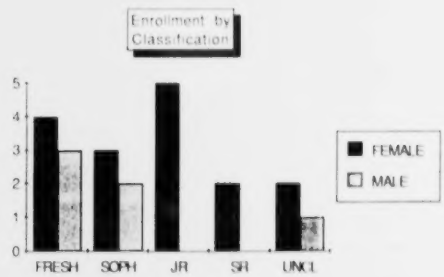
The tutoring log was also beneficial for the tutors, helping them to synthesize the tutoring session and review what had happened. Each student's progress in her or his math class was monitored. The staff knew when each student took a chapter test and the score. This information was important in order to provide encouragement and pinpoint problems.

Another significant aspect of our program was our weekly staff meetings. Here the problems of individual students were discussed and successful techniques shared. In these meetings we also reviewed basic math principles, discussed alternative approaches, and aired general concerns about the program.

Evaluation of Students

Of the twenty-two students enrolled in our course, sixteen were female and six were male. Four of the sixteen females withdrew from the course: one withdrew before the course began because she experienced so much math anxiety that she could not handle the course, and the other three were asked to withdraw because they did not attend the class. The charts below summarize the ethnicity and class level of the students enrolled. The average age of those who completed the course was thirty-five; the ages ranged from nineteen to sixty-five. Out of eighteen students, four were traditional-aged college students and the rest were returning students. Twelve of the eighteen students were enrolled in a math class they had not completed the previous semester.





At the end of the semester, seven students who were enrolled in the Math Learning Project received a B in their math courses, seven received a C, two received the RR grade (which meant that they had progressed far enough to continue in the course another semester), and two withdrew from their math courses. Of the four who withdrew from the Math Learning Project course, two received an F in their math classes and two withdrew from their math classes.

Course Evaluation

To evaluate the program, we collected information on the students enrolled in the course so that we could compare their previous performance in math with their performance during the semester they

were enrolled in this support course. We compared precourse and postcourse scores on an abbreviated version of the "Mathematics Anxiety Rating Scale" (Richardson and Woolfolk 1980). In addition, we had students complete a final evaluation questionnaire, a copy of which is in appendix E. In the first part of the questionnaire, we compared precourse and postcourse responses to selected open-ended questions concerning math learning. In the second part, we had students rate the effectiveness of specific topics or activities covered in the course. The third part covered students' overall feelings about the course and its structure.

Precourse and Postcourse Math Anxiety Rating Scale

An abbreviated form of the "Mathematics Anxiety Rating Scale" was given as a pretest and posttest. Here students rated their level of anxiety about a series of forty situations. On this scale a response of 1 indicates no anxiety and 5 indicates much anxiety; the situations relate to math test anxiety, math textbook anxiety, math class anxiety, algebra, observations of mathematics situations, decision problems, mixed numbers and graphs, and statistics.

The precourse and postcourse averages of scores for the participants were as follows:

Pretest: 3.17
Posttest: 2.58

The situations for which students most greatly reduced their anxiety from precourse to postcourse were the following:

Thinking about an upcoming math test one hour beforehand
Taking an exam in a math course
Talking about a problem to someone in your class who does well but not understanding what she or he is explaining
Asking your math instructor to help you with a problem that you don't understand

Students reduced their anxiety in all situations except one: having to use the tables in the back of a math book. The reduction ranged from 0.03 to 1.02.

Precourse and Postcourse Questionnaire

The first part of the final evaluation questionnaire consisted of five questions that the participants had answered during the initial interview:

1. Complete the sentence: The hardest part or topic in math was/is _____ and the easiest was/is _____.
2. Regarding your own learning style, what techniques do you use that work when you study math?
3. Complete the sentence: When I get stuck on a problem or don't understand the lesson in math, I _____.
4. Do you ever get anxious about math? When? What happens? How do you get over it?
5. What are your career or educational goals? How does math fit in? Have your goals changed since you began the semester?

Several observations could be made from the responses to these questions. Although students had changed their perceptions of what was easiest in math, there was no overall consensus. Word problems were considered the hardest for most students at the beginning of the semester and remained the hardest at the end of the semester. This is one area that needs continual review. In almost every case, the techniques students had identified as working for them at the beginning of the semester differed

from those identified as working for them at the end of the semester. It appears that students learned new or different techniques during the semester. On the precourse questionnaire, the majority of students had indicated that they would ask someone for help if they got stuck; two of them said they would do an easier problem until they got help or would review the example and then ask for help; and other students said they would generalize, quit, or walk around and hope the answer would come. On the postcourse questionnaire, only one person said she or he would ask for help first; half the respondents said they would try some other technique first and then ask for help, and half said they would try another technique, walk away, and then come back to it.

The postcourse responses indicated that students had learned alternative strategies to use when they got stuck and that they had become more resourceful. On both the precourse and postcourse tests, students indicated that they got anxious about math: in the precourse test, the responses were more generalized; in the postcourse test, those who specified when they felt anxious said that it occurred around testing. Finally, although career goals did not change, participants expressed a stronger desire to do better in math to meet their career goals.

Rating of Study Skill Techniques and Activities

The second part of the evaluation concerned the specific study skill techniques and activites that were designed for the class. Participants were asked to rate on a scale of 1 to 5, with 5 being most helpful, how helpful each technique was. The average for each is listed below:

Study Skill Technique or Activity	Average Rating
1. Recognizing the importance of carefully reading mathematics	4.94
2. Opportunity to share one-on-one with instructor/tutor	4.88
3. Working in groups with other people who have similar problems with mathematics	4.76
4. Learning to prepare for exams	4.58
5. Learning about the language of mathematics (math vocabulary cards, dictionary, vocabulary worksheets)	4.52
6. Problem solving using key words	4.41
7. Learning word problem designs (motion, work, coin/mixture, translations to math sentences)	4.41
8. Learning about math myths	4.29
9. Learning how to study mathematics	4.23
10. Learning about the situations and attitudes that contribute to math anxiety	4.17
11. Sharing your mathematics experiences with the class	4.11
12. Completing the "Mathematics Anxiety Rating Scale"	4.05
13. Problem solving using activities (logic puzzles, spatial puzzles)	4.00
14. Relaxation techniques	4.00
15. History of algebra	3.70
16. Writing the mathematics autobiography	3.70
17. Keeping a mathematics diary/journal	3.29
18. Learning about budgeting time	3.23

Rating of Course

The third part of the final evaluation consisted of a series of questions concerning the overall attitudes, structure, and environment of the course.

Students stated that at the beginning of the project they generally felt negative about taking math courses. Only a few participants indicated that they were hopeful or excited about being in the project. At the end of the semester, all students indicated that their attitudes had improved and said they felt more positive about math.

Understanding algebra and developing an organized approach to solving word problems were two concepts that students felt they had particularly been helped with. Students also cited time management as an important area.

A close and intimate atmosphere, the patience of the instructor, a supportive environment, encouragement to ask questions, and the interest in diagnosing why problems were occurring were all mentioned as positive aspects of the psychological environment of the project. The only recommendation for change was the suggestion to provide more counseling.

Students liked having different tutors who provided different teaching techniques. They also liked the slower pace of the course. Alternating lectures and problem-solving sessions were cited as being helpful. Also listed were lessons on setting up word problems, keeping cards for formulas, preparing for tests, and utilizing relaxation techniques, as well as activities using manipulatives. Techniques that were not helpful for some students were mixing ability levels on word problems and doing problems on the board.

Tutoring techniques that were considered helpful were showing how to organize materials, explaining step-by-step approaches, learning different ways of solving problems, providing positive reinforcement, and diagnosing why the problem was occurring. Also mentioned frequently were the tutors themselves, who were described as knowledgeable and patient. Techniques described as unhelpful for some students were having a question answered with a question and having a tutor tell a student, "You know how to do that."

Students said they had learned many specific study techniques that they would use again, such as preparing note cards, setting aside a certain time to study, going slowly, doing math every day, using the blackboard, asking questions, using relaxation techniques, and keeping a notebook.

Preparing the math autobiography taught students what caused their anxieties and negative attitudes, showed them that instructors had a substantial influence on their progress, and emphasized the importance of a positive attitude for success.

Whereas some students felt that the journals had helped them to understand their feelings and to see patterns in their behaviors, others said that the journals were not really helpful. One person felt that it might be more helpful if instructors responded to the journal entries.

On the whole, students liked best the unpressured, relaxed atmosphere; the positive attitude and support given; the feeling of success; the helpfulness of the tutors; the opportunity to share similar experiences with others; and the availability of assistance.

From all three parts of the final evaluations, students indicated that they had learned new study techniques and that they now applied a more positive approach when they had difficulty solving problems. They said they had learned from the activities we designed or incorporated from other programs. They liked the small-group atmosphere, noting that they had benefitted from being with others who had similar experiences, and said that the one-on-one assistance they had received was beneficial as well.

Conclusion

The results of the evaluation indicated that the course as designed was successful. The majority of the students successfully completed their arithmetic or algebra course and showed a reduction in anxiety on the "Mathematics Anxiety Rating Scale." The precourse and postcourse responses on the questionnaires indicated that students had learned new techniques that they were using and had a more positive attitude toward math. Finally, the overall comments on the course were consistently positive.

3

Lessons for the Math Learning Project Course

Math

The doors open.
Your mind is set
for a special time of devotion
to solving a problem.
One struggles in trying to
obtain the correct solution;
one thinks of quitting
yet the inner soul tells us
to don't quit.
Try, try, try again.
If you don't succeed,
try again.

Orlando Yniquez, a participant in the
Math Learning Project Course

The twelve formal lessons that we presented to our students are included in this chapter. Most of these lessons took place during one class period, but occasionally the lessons would last for more than one period. They were designed to run approximately one hour. Although each of these lessons is complete in itself, we sometimes used the same handout in more than one lesson.

When appropriate, we have included resources that we used to provide background in conducting a particular lesson. We have also included several possible ways to evaluate students' learning; you will want to select from these approaches.

This material would also be appropriate for workshops lasting less than twelve hours. In conducting shorter workshops, you should select those lessons which would best meet the needs of your students.

Lesson 1

Introducing Course, Staff, and Students, and Providing an Overview of Mathematics Anxiety

Rationale

It is important that the staff and students develop a caring relationship, because good interpersonal relationships provide the foundation for the success of this program. Learning names is the first step in this relationship.

A commitment to providing sufficient study time for mathematics is necessary to successfully complete a mathematics course.

An awareness of what mathematics anxiety means is necessary to deal effectively with the phenomenon.

Objectives

1. Each student and staff member will identify each member of the class by name.
2. Each student will prepare a weekly schedule showing class times, personal activities, and allotted study time.
3. Each student will read and discuss a description of math anxiety and identify with an item from the "Math Anxiety Bill of Rights."

Materials and Resources

Kogelman, S., and J. Warren. 1978. *Mind over math.* New York: McGraw-Hill.
Tobias, S. 1978. *Overcoming math anxiety.* Boston: W. W. Norton.

Handouts

"Course Information"
"Overcoming Math Anxiety and the Math Anxiety Bill of Rights"
"Guidelines for Managing Your Time"
"Participant Information"
"Weekly Schedule"

Instruction and Activities

1. Read the "Course Information" handout with students. The sample provided in this book is a sample only; prepare a similar one for use with your particular class. Answer questions and explain the course requirements and grading system.
2. Explain and lead the "Name Game":
   a. Tell something about yourself. Have each student do the same.
   b. Have students try to name one another by memory.
   c. Learn the names of the staff and students in class through this memory exercise.
3. Read the "Overcoming Math Anxiety" quotation together.

4. Ask students to read the "Math Anxiety Bill of Rights" silently.
   a. Ask each student to select one meaningful item to share with the entire group.
   b. Have each student recite with conviction her or his selected item.
   c. Lead a group discussion regarding the items.
5. Read aloud "Guidelines for Managing Your Time" and discuss the handout.
6. Ask students to complete the "Participant Information" and "Weekly Schedule" sheets.

Evaluation

1. Ask for a volunteer to identify by name as many of the participants as possible.
2. Ask participants each to share something that they learned about themselves or others during this class period.

Handout

Course Information

Univ 112 Section 2

Time: TTh 9-10 *or* TTh 4-5 *plus* one lab hour per week
Place: Jacobs Hall, Room 118

Instructors and Tutors (Office and Phone Numbers)
Margaret Scott, OH 156, ext. 1212 (office hours: 1:00–3:30 W)
Sandy Geiger, WH 34, ext. 2145
Sue Brown, Hardman 210, ext. 3136
Jacquie Kurland, Armando Torres, John Miller

Objectives

You will face your feelings about mathematics and how these feelings have affected your learning of mathematics. You will relearn or learn more about the basic operations of arithmetic with whole numbers, fractions, decimals, and percents and relearn or learn the fundamental ideas of algebra, operations with integers, solving equations, graphing, and, most important, how to use all of these in problem solving.

This course is intended to support your work in your math class (M100, M102, or M115).

General Information

You will keep an organized notebook (three-ring) in which lessons can be recorded chronologically. You may choose to use the same notebook that you use in M100, M102, or M115.

Throughout the semester, there will be handouts that you must complete and hand in. For each unit of work in your regular math class you will be asked to record important facts on a three-by-five-inch card to summarize the unit.

Grading will be based on class and lab attendance (three hours/week) and participation.

Laboratory "walk-in" hours: 8:30–5:00 MW; 10:00–3:45 and 5:00–6:00 TTh; 9:30–3:30 F
Place: Jacobs 118
Phone: ext. 5860

* * *

Remember: There are *no* stupid questions and there are *no* stupid mistakes.

Handout

Overcoming Math Anxiety and the Math Anxiety Bill of Rights

Overcoming Math Anxiety*

> *Doing math in the presence of intense anxiety is all but impossible. Once panic begins to take hold, normal functioning is impaired and skills necessary for learning and performing become inaccessible.*
>
> —S. Kogelman and J. Warren, *Mind over math*

Math Anxiety Bill of Rights†

I have the right to learn at my own pace and not feel put down or stupid if I'm slower than someone else.

I have the right to ask whatever questions I have.

I have the right to need extra help.

I have the right to say I don't understand.

I have the right not to understand.

I have the right to feel good about myself regardless of my abilities in math.

I have the right not to base my self-worth on my math skills.

I have the right to evaluate my math instructors and how they teach.

I have the right to view myself as capable of learning math.

I have the right to relax.

I have the right to be treated as a competent adult.

I have the right to dislike math.

I have the right to define success in my own terms.

---

* Excerpt from *Mind over math* by S. Kogelman and J. Warren. Copyright © 1978 by Stanley Kogelman and Joseph Warren. Reprinted by permission of Doubleday, a division of Bantam, Doubleday, Dell Publishing Group, Inc.

†From S. L. Davis, *Math anxiety bill of rights* (St. Paul: Continuing Education for Women, University of Minnesota, Minneapolis–St. Paul, n.d).

Handout

Guidelines for Managing Your Time

1. Set up a time for yourself each day when you can think about what you want and need to accomplish. Analyze your performance in the past. You can do this the first thing in the morning or the last thing before you go to bed. Or schedule your time once a week.
2. Write down everything you must do, using a notepad, a daily schedule, a calendar, or all three. Be specific. Don't just write, "Study math." Write down which pages you need to study.
3. Be organized before you begin an assignment. Have all your materials ready.
4. Decide what the most important thing is that you have to do and complete that job first before moving to the next. Give it your full attention. If you do not take the time to do it right, when will you have time to do it over?
5. Schedule study sessions for three or more hours, if possible. Inform your family and friends not to interrupt you during these sessions. Then be consistent and do not allow yourself to be interrupted.
6. Pick up a "Do Not Disturb" sign and place it outside the closed door of your office or workroom.
7. Plan a reward system for yourself. Remember to reward yourself *after* you have accomplished a goal, not *before*.
8. Use odd hours during the day for study. Anticipate when and where you could be forced to wait. Take class notes or reading material with you and review it.
9. Allow time in your plans for recreation, every day if possible. The time you spend doing this will help you study better.
10. Allow flexible time in your schedule so that you can deal with unexpected happenings.
11. Allow ten to fifteen minutes before each class for preview and the same amount of time following each class for review. Schedule a weekly review of notes and reading assignments for each course.
12. Do not be a crisis manager by trying to do too many things in too few hours (e.g., cramming for tests). Be realistic about your plans.
13. Be a skillful listener. We usually forget two-thirds of what we hear immediately after we hear it. This causes us to waste a lot of time. Write down information that you hear to save you time later.
14. Be considerate of other people's time.

Handout

Participant Information

Name _____

Address _____

Phone _____ Social Security Number _____

Major _____ Course time _____

Math class _____ Section _____

Starting point _____

Tutor _____

Course laboratory time _____

Alternate times _____ and _____

Handout

Weekly Schedule

Time	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
5:30 A.M.							
6:00 A.M.							
6:30 A.M.							
7:00 A.M.							
7:30 A.M.							
8:00 A.M.							
8:30 A.M.							
9:00 A.M.							
9:30 A.M.							
10:00 A.M.							
10:30 A.M.							
11:00 A.M.							
11:30 A.M.							
12:00 P.M.							
12:30 P.M.							
1:00 P.M.							
1:30 P.M.							
2:00 P.M.							
2:30 P.M.							
3:00 P.M.							
3:30 P.M.							
4:00 P.M.							
4:30 P.M.							
5:00 P.M.							
5:30 P.M.							
6:00 P.M.							
6:30 P.M.							
7:00 P.M.							
7:30 P.M.							
8:00 P.M.							
8:30 P.M.							
9:00 P.M.							
9:30 P.M.							
10:00 P.M.							
10:30 P.M.							
11:00 P.M.							
11:30 P.M.							
12:00 A.M.							
12:30 A.M.							
1:00 A.M.							

Lesson 2

Collecting Pretreatment Anxiety Data, Recording Feelings toward Mathematics, and Studying Mathematics

Rationale

One of the instruments used to measure mathematics anxiety will be given at the beginning of the program and again at the conclusion. These data will be analyzed for attitude change during the program.

Recognizing and describing feelings toward mathematics based on previous experiences is necessary before devising plans that deal with feelings and experiences.

Studying mathematics requires specific skills that are not always the skills used when studying other subjects. Students need to recognize these differences and apply proper techniques.

Objectives

1. Each student will complete the "Mathematics Anxiety Rating Scale," which will serve as a pretreatment measure for course evaluation purposes.
2. Each student will write a math autobiography to confront anxieties and other feelings about math.
3. Each student will maintain a math journal or diary to cope with feelings about math and to better understand difficulties in learning and performing math.
4. Each student will read and discuss suggestions and techniques for studying mathematics.

Materials and Resources

One three-ring notebook for each student

"Mathematics Anxiety Rating Scale," forty items. Adapted from Richardson, F. C., and R. L. Woolfolk. 1980. Mathematics anxiety. In *Test anxiety: Theory, research, and applications.* Ed. I. G. Sarason, 271–88. Hillsdale, N. J.: Lawrence Erlbaum Associates.

Handouts

"Math Autobiography"
"Math Journal or Diary"
"How to Study Mathematics"

Instruction and Activities

1. Administer the "Mathematics Anxiety Rating Scale" (twenty to thirty minutes).
2. Read and discuss the writing of a math autobiography. Ask each student first to share a personal positive experience in learning math and then to share a negative experience.
3. Read and discuss maintaining a math journal or diary. Each student should have a three-ring notebook to keep handouts, class notes, and math homework. Students are encouraged to make diary entries in this notebook in order to relate the entry to the math work they are doing in their course. Sample journal entries are in appendix D.

4. Ask students to read silently "How to Study Mathematics." Discuss item 1, "Choose a Good Attitude"; item 4, "Be Independent"; and item 8, "Be Neat and Accurate."

Evaluation

1. Ask students to turn in their math autobiography.
2. Ask students to write down a positive statement about their abilities to do mathematics.

Handout

Math Autobiography

Writing about your own history with math can help you get in touch with your anxieties and other feelings about math. Looking back helps you see how ideas were formed and how methods were developed to deal with math.

Write down your history with mathematics. Doing so will help you gain insight into why you feel the way you do about mathematics. Here are some ideas and questions to trigger your memory:

Recall any positive experiences you had in learning math.
When do you remember doing well in math?
What bad experiences do you remember?
Did you ever like math? When did that feeling change?
Who helped you at home with math?
What did your father think about math? your mother?
What did your brothers and sisters think about math? your friends?
What do you remember about studying math in elementary school? in junior high school? in high school? in college?
What do you remember about individual math teachers you have had?
What did people say to you when you made a mistake in math or when you did exceptionally well?

You may wish to start with a time line by writing down events and ideas and then elaborating on them. Add information as you remember events.

Due in two weeks.

Handout

Math Journal or Diary

A math journal or diary is a place to work out feelings about math and to better understand difficulties you have in learning and performing math. It is a good place to record math-related activities we do, new skills we try, new things we learn, or frustrations we encounter.

Spend a little time each day looking at the math-related activities you have done. Write about how you felt doing or not doing the math and write about what you learned from the experience.

Due at end of semester.

Handout

How to Study Mathematics*

Many people have trouble learning mathematics because they never develop the particular study habits that are conducive to success in mathematics. If you practice the following suggestions, you will develop good study skills.

1. Choose a Good Attitude.

Decide that you are going to do your mathematics in the most successful way possible and choose a good attitude to accompany your efforts. Many students do not realize that this choice is even available.

Be selective about the environment in which you choose to study math. Decide what methods work and what time works.

2. Read Carefully and Deliberately.

The way you should read a mathematics book is quite different from the way you read a history book, a newspaper, or a novel. In mathematics you must read slowly, absorbing each word. It is sometimes necessary to read a discussion or problem many times before it begins to make sense to you. In some types of reading, such as a novel, it is desirable to skim and read rapidly because there are usually a few thoughts sprinkled among many words. In reading mathematics, however, each word or symbol is important because there are many thoughts condensed into a few statements.

3. Think with Pencil and Scratch Paper.

Have pencil and scratch paper ready and use them when you read and study mathematics. Test out on paper the ideas that the author is discussing. When a question is proposed, try to answer it before going on. Even though an example may be worked out completely in the text, work it out for yourself on scratch paper. Doing so will help to clarify the ideas and procedures in your mind before you start the exercises. After you have read and reread a problem carefully, if you still do not see what to do, do not just sit and look at it. Get your pencil going on scratch paper and try to "dig it out." If, in attempting to solve a problem, you have nothing written on paper, then certainly you have not yet exerted enough effort to justify seeking help.

4. Be Independent.

Try to complete each lesson without assistance. If you seek help needlessly, from your tutor, classmates, or math learning center staff members, you will not gain the maximum benefit from your work. It takes exercise to become strong. You cannot do it through someone else's exercise. You must, however, ask questions when necessary. Sometimes little things cause considerable confusion. Do not be afraid that your question may sound stupid. The only "stupid" action is to fail to ask about a topic that you have really tried to grasp and still do not understand. Some people seek help too soon,

* Adapted from J. Threadgill-Sowder, N. McKenna, A. Besserman, and D. Regan, *How to study mathematics* (Malta, Ill.: Kishwaukee College, 1983).

and some wait too long. You will have to use good common sense in this matter.

Learn what you can about your own particular learning style.

5. Discuss Ideas and Concepts.

Discuss math ideas on a regular basis with your tutor or a friend. Better yet, try teaching a particular idea to another student. Explaining something to someone else is often the best way to learn it. Sometimes you will have a feeling about a problem or procedure that you cannot explain but that you know is correct; this, too, is okay.

6. Persevere and Take Time to Reflect.

Do not become frustrated if a topic or problem completely baffles you at first. Stick with it! An extremely interesting characteristic of learning mathematics is that at one moment the learner may feel totally at a loss and then suddenly have a burst of insight that enables her or him to understand the topic or problem perfectly. If you do not seem to be making any progress after working on a problem for some time, put it aside and attack it again later. Many times you will then see the solution immediately, even though you have not been consciously thinking about the problem in the meantime. There is a tremendous sense of satisfaction in having been persistent enough and creative enough to independently solve a problem that had given you a great deal of trouble.

To learn mathematics well, you must take time to do some reflective thinking about the material covered during the previous few days or weeks. It takes time for some ideas in mathematics to sink in. You may have to live with them awhile and do reflective thinking about them before they become a part of you. You must continue to use such concepts to "keep" them.

7. Concentrate on Fundamentals and Learn the Vocabulary.

Do not try to learn mathematics by memorizing illustrative examples. You will soon become overwhelmed by this approach, and the further you go the less successful you will be. All mathematics is based on a few fundamental principles and definitions. Some of these you must memorize. But if you concentrate on these fundamentals and try to see how each new topic is just a reapplication of them, very little additional memorization will be necessary. It may be helpful whenever you come across a new word in the text to (a) write the word on a note card; (b) write the meaning of the word on the back of the card; (c) practice writing the word, reading the definition, and using the word in some sentences; and (d) set up a definite study time for reviewing new vocabulary words.

8. Be Neat and Accurate.

Being neat and accurate are habits that will prevent many headaches in any field of endeavor. Everyone must deliberately practice neatness and accuracy before these become a habit. Keep your work organized. Try to organize the material as if it were intended for the printer. Have a special section in your notebook for mathematics. Keep today's homework paper in the same place in this section every day so that you can turn to it immediately.

9. Take Time to Do Your Work and Do It on Time.

You must do your homework regularly and make up the work you miss when you are absent. Mathematics is built vertically, like a tall building in which the upper floors are supported by the lower ones. You cannot afford to leave gaps in this important foundation. Do not wait until the last minute to do your work and then rush through it. If you spend just enough time on your lesson to get the answers and do not take time to really understand the underlying principles, you will soon become

confused. In doing your homework exercises, be aware of the reasons for the steps you go through. Mathematics is a lot of fun as long as you are on top of it and understand what is going on; otherwise, it is very frustrating.

Learning mathematics is not an activity for the intellectually lazy. It requires a strong, steady effort. There is no other successful way to learn the subject. Neither is mathematics a spectator sport; you must become actively involved. Do not expect to sit idly by and watch your teacher do the work. That may keep your teacher in good condition, but it will not do you much good. It is easy to be fooled into thinking you do not need to practice because it looks easy for the teacher. Remember that when the instructor is making explanations she or he is thinking out loud for you. You need to practice thinking, too. So take time in your own study to think and solve as many exercises as time will allow.

Do not forget that the main objective of the written homework is to learn the ideas of mathematics by putting them into practice. Sometimes the written homework also has the objective of showing how much you have learned.

There will be no extra compensation given for working hard or for conscientiously doing your homework. You are expected to do those things as a matter of course. The reward you get will be the mathematics that you learn and the feeling of satisfaction you gain because of the learning.

Lesson 3

Examining Math Myths

Rationale

Attitudes toward mathematics may be based on inaccurate information. Comparisons between attitudes and facts may help reduce anxiety.

Objectives

1. Each student will recognize myths about mathematics.
2. Each student will examine how these myths have contributed to her or his anxiety about mathematics.

Materials and Resources

Kogelman, S., and J. Warren. 1978. *Mind over math.* New York: McGraw-Hill.

Handouts

"Math Myths"

Instruction and Activities

1. Distribute "Math Myths" and ask students to check the statements they believe are true. Tally the responses by a show of hands.
2. Discuss the twelve myths, paying particular attention to those items that a number of students believed were true. (Use Kogelman and Warren, pp. 30–42, as a resource.) The answer for all twelve is "false."

Evaluation

1. Ask students which myths have contributed to their anxiety about mathematics.
2. Ask students to suggest ways to help others recognize that these statements are myths.

Handout

Math Myths*

Indicate which of these statements are true by placing a check before the true statements:

1. Men are better in math than women are.

2. Math requires logic, not intuition.

3. Math is not creative.

4. You must always know how you got the answer.

5. There is a best way to do a math problem.

6. It's always important to get the answer exactly right.

7. It's bad to count on your fingers.

8. Mathematicians do problems quickly, in their heads.

9. Math requires a good memory.

10. Math is done by working intensely until the problem is done.

11. Some people have a "math mind" and some do not.

12. There is a magic key to doing math.

* Excerpt from *Mind over math* by S. Kogelman and J. Warren. Copyright © 1978 by Stanley Kogelman and Joseph Warren. Reprinted by permission of Doubleday, a division of Bantam, Doubleday, Dell Publishing Group, Inc.

Lesson 4

Discussing Mathematics with Others and Developing a Mathematics Vocabulary

Rationale

Using mathematics skills depends on understanding and using the vocabulary associated with the subject. Discussing concepts and providing explanations to others are ways of strengthening one's understanding of mathematics.

Objectives

1. Students will define mathematical words they recognize and use them comfortably and accurately.
2. Students will define mathematical words they want to understand.
3. The group will develop a dictionary of mathematical terms to be used in the course.

Materials and Resources

Index cards in two colors (such as yellow and green), one-hole punched
Metal ring to assemble cards for dictionary

Handout

"How to Study Mathematics," from Lesson 2

Instruction and Activities

1. Read and discuss two items from "How to Study Mathematics": item 5, "Discuss Ideas and Concepts," and item 7, "Concentrate on Fundamentals and Learn the Vocabulary."
2. Ask each student to select one mathematics vocabulary word or concept that she or he knows and understands. Have the student write the word or concept on one side of a yellow card and write the definition and an example on the other side.
3. Have each student share a vocabulary word with the group and discuss the definition and example.
4. Ask each student to write on a green card, a math word or concept she or he does not understand.
   a. Ask students to turn in the vocabulary cards.
   b. Have students test one another on the vocabulary cards.
5. Assemble a mathematics dictionary by alphabetizing all the green and yellow cards for use by the class as a reference. Encourage students to add concepts or words to the class dictionary as they come across them. Whenever a student can define a word on a green card, she or he should remove the green card and replace it with a yellow card defining that word. Suggest that each student maintain a personal list of these words and concepts in her or his math notebook.
6. Periodically review the vocabulary with the tutors.

Evaluation

1. Ask students to turn in vocabulary cards weekly.
2. Ask students to explain a new vocabulary word and concept each week.
3. Ask students to create a handout explaining a concept step by step. A sample of an exercise is in appendix D.

Lesson 5

Reading Mathematics and Following Through on Organizing for Studying Mathematics

Rationale

The language of mathematics uses words with precise meanings. To successfully read mathematics, one must know these meanings and understand the context in which the words are used.

Mathematics requires careful and deliberate reading. Students must read what is written and not interject expectations.

A daily encounter with mathematics can include overview, review, cross-referencing, vocabulary, or history. This encounter is something aside from normal mathematics study and homework.

Objectives

1. Each student will experiment with suggestions on improving mathematics reading skills.
2. Each student will examine times allocated for studying mathematics.

Materials and Resources

Index cards and scratch paper

Kogelman, S., and J. Warren. 1978. *Mind over math*, 163–68. New York: McGraw-Hill.

Lamb, C., and N. Zeliavi. 1984. Elementary mathematics vocabulary. In *Diagnostic and prescriptive mathematics issues, ideas, and insights*. Ed. G. Bright and J.M. Hill, 58–61. Kent, Ohio: Research Council for Diagnostic and Prescriptive Mathematics.

Handouts

"How to Study Mathematics," from Lesson 2
"Math Reading Test"
"Answers to Math Reading Test"

Instruction and Activities

1. Review vocabulary from the previous lesson.
2. Discuss requirements for successfully reading mathematics: From "How to Study Mathematics," read item 2, "Read Carefully and Deliberately."
3. Provide an example to illustrate careful reading, such as the following:

   There are 4 Arabians and 3 thoroughbreds in the pasture.
   Are there more Arabians than there are horses?

4. Suggest that students use two index cards to avoid visual overload by blocking out all but the text they need to concentrate on. Demonstrate this method and have students practice. Suggest that students also highlight important material.

5. Read and discuss Kogelman and Warren's suggestions for reading a math textbook.
6. Read and discuss item 3, "Think with Pencil and Scratch Paper," from "How to Study Mathematics." Suggest that students write big.
7. Distribute the "Math Reading Test." Allow students to answer the first few items. Discuss the first three items and request that students write the solutions on the chalkboard as they solve the problems.
8. Whenever possible check with students on their success in maintaining a schedule that allows for studying mathematics each day. Share problems and results. Tutors can review each student's schedule, skim the journal and notebook, and discuss ways of making the study of mathematics more organized.

Evaluation

1. Have students identify the most important technique they learned about reading mathematics.
2. Ask students to list a step-by-step approach for reading mathematics.
3. Ask students to identify one new technique they will use when reading mathematics.

Handout

Math Reading Test*

1. Does England have a fourth of July?

2. If you had only one match and entered a room where there was a lamp, and oil heater, and some kindling wood, which would you light first?

3. A woman gave a beggar 50 cents. The woman is the beggar's sister, but the beggar is not the woman's brother. Why?

4. Is it legal in North Carolina for a man to marry his widow's sister?

5. A garden had exactly 50 different kinds of flowers, including 10 kinds of roses, 3 kinds of sweet peas, 2 kinds of alyssum, 5 kinds of carnations, 3 kinds of zinnias, 8 kinds of poppies, 4 kinds of snapdragons, 5 kinds of gladiolus, and 6 kinds of phlox. How many different kinds of flowers did the garden have?

6. A rooster is sitting on the peak of a roof and lays an egg. Which way does the egg roll: to the right or to the left?

7. Abbott, Baker, and Casper are a detective, and an entomologist, and a farmer, although not necessarily in that order. Abbott was the proud mother of healthy twins yesterday. Casper has a deathly fear of insects and will not even get close enough to one to kill it if she sees it. The farmer is getting worried because she and her husband are getting old and will not be able to run the farm for too many more years, and she has no children. Casper, unmarried, especially likes to date brunettes. What is the occupation of each of the three women?

8. Three quarrelsome men registered at a hotel and paid $30 for a suite of rooms, each man contributing $10. The clerk discovered later that he should have charged them only $25 for that suite, so he gave the bellboy $5 to return to the men. Remembering how the men had quarreled when they registered, the bellboy thought they would quarrel too much about how to split up the $5 refund, and so he kept $2 and returned $3 to the men. Now each man had paid $10, less $1 refund for the room ($9 x 3 men = $27). But $27 + $2 kept by the bell boy equals only $29, and the original charge was $30. What happened to the other $1?

* Adapted from Jean Smith, Math Clinic, Wesleyan University, Middletown, Connecticut.

Handout

Answers to Math Reading Test

Possible solutions

(Note this is a "reading" test; don't let the numbers fool you. *Think* and *picture* rather than compute.)

1. Yes, but it is not a holiday.
2. the match
3. The beggar is her sister.
4. No. If his wife is a widow he would be dead.
5. 50
6. Roosters don't lay eggs.
7. Abbott—entomologist; Baker—farmer; Casper—detective
8. men paid 9 x 3 = $27
   Cost of room $25 + $2 clerk kept

Lesson 6

Introducing Problem Solving

Rationale

The National Council of Teachers of Mathematics (1980) recommended problem solving as the focus of mathematics instruction.

Students need to develop problem-solving strategies that can be applied to real-life situations. Students may be using many of these strategies in real-life situations but not applying them to math textbook problems. Problem-solving strategies can be taught, providing students with a wider range of options when applying math skills.

Objectives

1. Students will consider different strategies to use in solving problems.
2. Students will analyze their present personal skills in solving problems.
3. Students will study and apply an attack plan for solving word problems.
4. Students will discuss and learn a list of key words used in word problems.

Materials and Resources

Two dozen toothpicks and nine wooden blocks per student

Downie, D., T. Slesnick, and J. Stenmark. 1981. Toothpick puzzles. In *Math for girls and other problem solvers*, 67. Berkeley, Calif.: The Regents of the University of California.

Handouts

"Toothpick Puzzles"
"Problem-Solver Inventory"
"Auxiliary Skills: Problem Solving"
"Key Words"
"EQUALS Problem-Solving Strategy List"

Instruction and Activities

1. Distribute "Toothpick Puzzles." Ask students to solve the problems using blocks and toothpicks and to identify the strategies they used.
2. Ask students to complete the "Problem-Solver Inventory." Discuss strategies that they are already using and how these skills can be applied to solving mathematical problems.
3. Ask students to read and discuss "Auxiliary Skills: Problem Solving."
4. Ask students to read and discuss "Key Words." Define words that may cause confusion by using simple examples.
5. Ask students to read the "EQUALS Problem-Solving Strategy List" for suggestions on what to do when they get stuck. When students are having difficulty during lab time, refer them to these suggestions.

Evaluation

1. Have students complete the handouts.
2. Ask students to write a brief summary of their strengths as a problem solver.
3. Throughout the semester, ask students to verbalize their problem-solving strategies.

Handout

Toothpick Puzzles*

1. Use 24 toothpicks to make this figure.

   a. Remove 4 toothpicks and leave 5 squares.
   b. Remove 8 toothpicks and leave 4 squares.
   c. Remove 8 toothpicks and leave 2 squares.
   d. Remove 8 toothpicks and leave 3 squares.
   e. Remove 6 toothpicks and leave 3 squares.

2. With 9 toothpicks, make this figure.

   a. Remove 2 toothpicks and leave 3 triangles.
   b. Remove 3 toothpicks and leave 1 triangle.
   c. Remove 6 toothpicks to get 1 triangle.
   d. Remove 4 toothpicks to get 2 triangles.
   e. Remove 2 toothpicks to get 2 triangles.

3. Use 15 toothpicks to make this figure.

   a. Remove 3 toothpicks and leave 3 squares.
   b. Remove 4 toothpicks and leave 3 squares.
   c. Remove 5 toothpicks and leave 3 squares.
   d. Remove 4 toothpicks and leave 2 squares.
   e. *Move* 3 toothpicks and make 4 squares.

4. Construct this figure using 17 toothpicks.

   a. Remove 5 toothpicks and leave 3 squares.
   b. Remove 6 toothpicks and leave 2 squares.

5. Use 12 toothpicks to make this figure.

   a. Remove 2 toothpicks to leave 2 squares.
   b. *Move* 4 toothpicks to make 10 squares.
   c. *Move* 2 toothpicks to make 7 squares.
   d. *Move* 3 toothpicks to make 3 squares.

* Adapted from handouts provided by the EQUALS Program at the Lawrence Hall of Science, University of California, Berkeley.

Handout

Problem-Solver Inventory*

Directions: Place a check mark by those personality traits that you believe describe you.

_____ Flexible
_____ Open–minded
_____ Curious
_____ Enjoy trying new things
_____ Don't make hasty judgments
_____ Like to learn new things
_____ Like challenges
_____ Like to figure out things myself rather than having someone else tell me

Now do the same thing with this list of skills:

_____ Reading carefully
_____ Looking up new words in a dictionary
_____ Comparing two or more things, noticing similarities and differences
_____ Noticing details
_____ Separating important from unimportant information
_____ Planning ahead
_____ Breaking complex problems into simple parts
_____ Organizing information

Finally, here is a list of experiences. Check any of these which are similar to ones you have had.

_____ Read, interpreted, and carried out directions accompanying a sewing pattern
_____ Designed, planted, and tended a vegetable garden
_____ Built a model
_____ Assembled a piece of equipment
_____ Solved a photography problem related to lighting, depth of field, or shutter speed
_____ Solved jigsaw puzzles
_____ Played board games

Look back over these checklist. Very probably you have checked several items in each category. Since the lists contain personality traits, skills, and experiences that are useful in problem solving, you are not starting totally without skills. You have already developed some problem-solving abilities; now you are going to build on these to learn more.

* Adapted from J. T. Ray and C. A. Oxrieder, *Your number's up* (Reading, Mass.: Addison-Wesley, 1982), 70–72.

Handout

Auxiliary Skills: Problem Solving

Rule of thumb: Always start assignments ahead of time, if possible. Problem solving can take days, weeks, and yes, even months or years.

Attack Plan for Word Problems

1. Read the problem carefully at least twice. Can you guess or estimate the answer? Try.

2. List the information given (the data). Draw a picture or make a chart, table, or diagram relating all the information given that you think is important.

3. Decide what is asked for. What is the missing information? Most likely, it will be located in the question. Look in the sentence with the question mark or after the word *find*.

4. Write a sentence using words describing the relationship between the known and the missing information.

5. Assign a variable name to the missing information, and write down that name.

   Example: Let X represent the length of the rectangle.

6. Form an equation. Perhaps a formula is necessary.

7. Substitute known values into the equation.

8. Solve the equation.

9. Check your solution with the wording of the problem to make sure your solution makes sense. Check your solution mathematically by substituting your answer back into the original equation and evaluate the original equation. Have you answered the question? Make sure to include the correct unit of measurement.

* * *

In short:
1. Read and guess.
2. List and draw or chart the information given.
3. Determine the missing information.
4. Write a sentence.
5. Assign a variable name to the missing information.
6. Form an equation.
7. Put known values in.
8. Solve for the unknown.
9. Check. Answer the question.

Handout

Key Words

Addition	*Subtraction*	*Multiplication*	*Division*	*Equals*
add	subtract	multiply	divide	is
sum	difference	product	quotient	
plus	minus	times	per	
increased by	less than	twice, thrice,		
more than	decreased by	four times,		
	take from	and so on		
		of		

When You Get Stuck

1. Think of every formula or definition that might be relevant to the problem. Write them down.
2. Work backwards. Ask yourself, "What do I need to know in order to get this answer?" (provided that the answer is given somewhere).
3. Relate the problem to a similar example from your textbook or notes.
4. Solve a simpler case of the problem using large or small numbers, and then follow this example as if it were an example from the text.
5. Break the problem into simpler problems. Work part of the problem and see if it relates to the whole.
6. If you are making *no* progress, take a break and return to the problem later.

Handout

EQUALS Problem–Solving Strategy List*

Check those strategies you have used. In the blank indicate the problem type or situation in which you used the strategy. Refer to the list when you get stuck. Update when you are successful.

Move It Around

_____ Act it out. _____

_____ Use blocks or beans. _____

_____ Sort or classify the parts. _____

_____ Measure it. _____

Picture It

_____ Close your eyes and listen to the story words. _____

_____ Draw a picture or diagram. _____

_____ Use a grid or array. _____

_____ Make a tally or graph. _____

_____ Make a list. _____

_____ Find things that are alike or different in the problem. _____

Look at the Problem

_____ Follow the directions. _____

_____ What is the first thing the problem tells? _____

_____ What is the next thing the problem tells? _____

_____ Is there anything else you need to know? _____

_____ Is there information you do not need? _____

Watch Your Language

_____ Put it into your own words. _____

_____ Does it sound like a problem you have done before? _____

* Adapted from handouts provided by the EQUALS Program at the Lawrence Hall of Science, University of California, Berkeley.

_____ Does the problem make sense? _____

_____ Talk to someone else about the problem. _____

And . . .

_____ Practice guessing at the answer before working the problem. _____

_____ Check your arithmetic. _____

_____ Use a calculator. _____

_____ Read the problem again with the answer finished. _____

_____ *Think about it for a while, and then try again.* _____

Lesson 7

Reducing Test Anxiety

Rationale

Many math-anxious students experience test anxiety. By using positive self-thoughts and relaxation techniques, they can learn to control their anxiety. People cannot be relaxed and anxious or tense at the same time. If students can learn to relax themselves, they can get rid of the anxiety and tenseness.

Objectives

1. Students will recognize the causes of test anxiety.
2. Students will experience systematic relaxation as a way of reducing anxiety.
3. Students will learn to change negative self-thoughts into positive thoughts about themselves.

Materials and Resources

Instructor sheets

"Reducing Test Anxiety"
"Systematic Relaxation Script"

Instruction and Activities

1. Go through the "Reducing Test Anxiety" activity with students.
2. Introduce systematic relaxation as a technique to use to get rid of anxiety and tenseness.
   a. Go through the "Systematic Relaxation Script" slowly.
   b. At the conclusion, ask students to share where they went, what it was like, and how they felt while they were there.
   c. Conclude with this discussion:
   Systematic relaxation is something that you can practice every night before you go to bed. Go through your body systematically, tensing and relaxing all parts. Remember to focus on breathing deeply. The more you practice, the easier it will be for you to relax. If you begin to get tense when you are studying mathematics, take a few moments to systematically relax your body.

   You may notice that a particular part of your body feels tense. Some people carry tenseness in their necks, others in their backs. Some people notice the tenseness in their stomachs. A shortcut to relaxing is to focus your breathing on a particular area. If you feel tense in your neck, close your eyes and focus your breathing through your neck. Feel your breath soothe the tense area. Again, this takes practice. The more you do it, the easier it will be for you to relax yourself.

Evaluation

1. Ask students to keep track (in their journals or diaries) of negative statements that they make. Have them change the negative statements to positive ones.
2. Ask students to practice the relaxation technique during the week and report on its effectiveness.

Instructor Sheet

Reducing Test Anxiety

[Say to students:]

Practically everyone experiences some test anxiety. Feeling a little anxious is normal and can even be helpful. It provides you with the stimulation to put forth a strong effort. It keeps you on your toes. Test anxiety is bad when it becomes debilitating.

The test itself does not cause test anxiety. It is your reaction to the test that causes the anxiety. If you can change your reaction, you can rid yourself of the anxiety. We are going to do an exercise to illustrate this point.

[Ask students to do the following:]

1. Think of a recent test experience you had in which you were anxious. (If you have not had a test, think of any experience you had in which you felt anxious.)
2. Close your eyes and picture yourself in that situation. Focus on the thoughts that are going through your mind.
3. Open your eyes. On a piece of paper, write down the thoughts or words that you said to yourself.
4. Turn in the papers.
5. Close your eyes while I read some statements to you. Repeat the statements to yourself.

[Read only negative statements for this activity.]

I know I won't pass.
I'm going blank.
I hate math.
I don't know anything.

[Read the statements slowly. Give students time to repeat the statements to themselves. Conclude the activity with the following questions and discussion:]

1. How did you feel when you were repeating the negative statements?
2. How do negative statements increase anxiety?
3. How could you change negative statements into positive statements to reduce anxiety?

[Discussion:]

If you focus all your energy on negative thoughts, it will be very difficult for you to be able to do well on tests. Picture a basketball player at the free-throw line saying, "I know I'm going to miss, I always miss, I can't do it." Or the marathon racer saying, "I'll never make it, I'm going to quit, I can't do it." If that is what is going through their minds, they will never achieve their goals. The same thing goes for you as you take tests. So that you can succeed, you need to focus your energy on positive thoughts, such as: "Relax, I'm going to do the best that I can. This is just one test. Let's see how much I know." You will be surprised at how much better you can do when you think positively.

Here's another suggestion: When you are feeling anxious, try the "best friend" technique. Treat yourself as you would your best friend. You would not tell your best friend, "I know you're going to

fail, you always go blank, you aren't very good at math." You would encourage your friend to do well and you would give her or him positive support. So tell yourself what you would tell your best friend and you will feel better.

Instructor Sheet

Systematic Relaxation Script

[Read the following slowly to students, pausing between each sentence:]

Close your eyes. Lean back and get comfortable in your chair. Breathe deeply. Feel your body relax as you deeply and slowly exhale. Breathe in through your nose and exhale through your mouth. Relax your entire body. Focus on the rhythm of your breathing. Let go of all the tension in your body. Clench your fists as tightly as you can. Breathe deeply, and slowly tighten your forearms. Make them tighter, breathe deeply. Tighten your biceps; make them tighter. Tighter. Feel the tension. Now relax. Relax your arms and hands, breathing deeply. Feel the difference as you breathe deeply.

Tighten your forehead. Make your forehead wrinkled, tighter and tighter. Now relax your forehead. Clench your teeth as tightly as you can. Clench them tighter, tighter. As tightly as you can. Now relax your jaw. Feel the tension go. Breathe deeply. Let your head fall back. Stretch your neck as far back as you can. Farther. Farther. Bring your head up. Relax. Breathe deeply. Feel your body relax.

Shrug your shoulders. Shrug them as high as you can. Feel the tightness. Now relax. Let your shoulders relax. Breathe deeply. Slowly. Tighten your chest muscles. Tighten them as hard as you can. Tighter. Tighter. Tighter. Now relax, breathing slowly and deeply. Notice the difference. Tighten your stomach muscles. Tighten them more. Make them as tight as you can. Now relax. Breathe in and out. Breathe deeply.

Tighten your hips. Make them as tight as you can. Tighter. Tighter. Feel the tension. Now relax. Breathe deeply through your entire body. Tighten your thighs. Tighter. Tighter. Tighter. Make them as tight as you can. Relax. Breathe deeply. Notice the difference. Tighten your calves by pressing your feet to the floor. Press harder. Harder. Feel the tightness. Now relax. Breathe deeply. Feel the difference. Curl your toes. Make them tighter. Tighter. Tighter. Now relax. Breathe deeply through your entire body. Feel the air through your head, down your arms, into your chest, through your stomach. Through your thighs, down to your calves and feet. Feel your whole body relax.

While you are relaxing and breathing deeply, picture yourself in your favorite environment—a place where you feel totally at peace. Notice where you are. What things do you see in your favorite place? Notice the colors. What sounds do you hear? What smells do you smell? Touch some things around you in your favorite place. How do they feel? Picture yourself getting up and going for a walk. What do you see? What do you hear? What do you smell? Notice the calmness that you feel inside. Notice the strength that you feel.

Take a deep breath. Spend a few minutes relaxing in this environment as you take it all in with all of your senses. [Allow two to three minutes.]

When you have a good feeling for your total environment and how you feel while you are there, you may slowly open your eyes.

Lesson 8

Evaluating Individual Students' Progress to Date and Evaluating Tutoring

Rationale

It is important to stop and look at individual students' progress midway through the semester. It is also important at this time to give tutors feedback on their performance.

Objectives

1. Students will evaluate their own progress.
2. Instructors will evaluate student progress.
3. Students will evaluate tutor's performance.

Materials and Resources

Student notebook and journal

Handouts

"Sample Agreement"
"Student Evaluation of Tutors"

Instruction and Activities

1. Set up individual meetings with each student.
2. During the meetings, do the following:
   a. Ask the students to share their perception of the progress they are making in math learning. They should use their notebooks and journals during the meeting to help them remember significant incidents.
   b. Take notes for sharing information with other instructors or tutors as appropriate.
   c. Encourage students to freely express their ideas and concerns.
   d. Alert students who are not satisfactorily meeting the requirements of the course (i.e., attending class or completing homework assignments). Try to find out the reasons why. If students seem willing to improve, set up agreement forms for them to sign (see handout).
3. At the end of the conference, give each student the tutor evaluation form to fill out anonymously.
4. Provide an envelope for students to leave the completed tutor evaluations.

Evaluation

1. Review and discuss information from the private conferences with appropriate instructors and tutors.
2. Share with tutors the results of students' evaluations of tutors.

Handout

Sample Agreement

I, _____, have the following unexcused absences from Univ 112–2 lab:

Since the grade for this course depends on attendance alone, I understand that I must make up all but two of these absences in order to get an S in this course.

Signed _____

Date _____

Contract

I will attend the following lab sessions to make up my absences:

Day/Date **Time**

Handout

Student Evaluation of Tutors

Please help us in our tutor training program by answering the following questions based on your experiences with this tutor. Evaluations are anonymous. Place the completed form in the envelope provided.

Tutor: _____

	Yes	No	N.A.*
1. Did the tutor help you feel at ease?	___	___	___
2. Did the tutor try to diagnose your difficulties?	___	___	___
3. Did the tutor use an effective strategy to help you?	___	___	___
4. Did the tutor help you feel successful?	___	___	___
5. Would you feel comfortable going to this tutor for help in the future?	___	___	___

6. What are your tutor's strengths?

7. What suggestions do you have for improvements?

8. What other comments do you have?

*Not Applicable.

Lesson 9

Translating English into Mathematics

Rationale

In Lesson 6 students received a problem-solving handout ("Key Words") that included a vocabulary list of synonyms for *addition, subtraction, multiplication, division,* and *equals.* Practicing direct translation from simple English sentences into mathematical expressions is the second step to strengthening problem-solving techniques, exhibiting a direct link between the real world and mathematics.

Objectives

1. Students will each complete two exercises on translating English into mathematics.
2. Students will add to list of synonyms as needed.

Materials and Resources

Handouts

"Auxiliary Skills: Problem Solving," from Lesson 6
"Translation Exercise 1"
"Answers to Translation Exercise 1"
"Translation Exercise 2"
"Answers to Translation Exercise 2"

Instruction and Activities

1. Have students locate their copies of "Auxiliary Skills: Problem Solving." Review the list of synonyms together.
2. Distribute "Translation Exercise 1."
   a. Do the first few together on the blackboard with the class. Encourage left-to-right, "word-for-word" translations, whereby the student circles words from the synonym list and replaces them with the correct math symbol.
   b. Point out danger areas, such as *less than* or *subtracted from,* and the use of words such as *of* aside from their math usage, as in *sum of.*
   c. Have students complete "Translation Exercise 1" and share their solutions aloud.
3. Distribute "Translation Exercise 2." Have students complete the exercise and share their solutions aloud.
4. Ask students to write their own English expressions and have a partner write the math translation.

Evaluation

Ask each student to write down five English expressions and make an answer key with the correct math translation. Then have students exchange their lists of expressions and write down the correct math translations for each expression listed.

Handout

Translation Exercise 1

Using your guide for synonyms for *addition*, *subtraction*, *multiplication*, *division*, and *equals*, translate the following sentences into mathematical equations or expressions. It is not necessary to solve these equations at this time. Add to your list of synonyms when possible.

Variable assignment: Let *X* represent the unknown number in each sentence. Let *Y* represent the second number if necessary.

1. The sum of two numbers is 15.
2. Four times a number is 12.
3. Three more than 4 times a number is 15.
4. The product of 6 and a number is 36.
5. Four less than 40 is what number? (*Hint:* Reverse the numbers.)
6. The product of 6 and a number is 4 less than 40.
7. The difference between 25 and a number is 15.
8. The quotient of 100 and a number is 10.
9. The difference between 25 and a number is the quotient of 100 and that same number.
10. Fifty percent of a number is 12.

Tricky Ones

11. One number exceeds 17 by 5.
12. Twelve less than what number is 2?
13. The square of a number is its double.
14. Decrease a number by 7 and then increase this difference by 9.
15. The sum of 3 consecutive integers is 3 times the middle integer.

Handout

Answers to Translation Exercise 1

1. $X + Y = 15$
2. $4X = 12$
3. $3 + 4X = 15$
4. $6X = 36$
5. $40 - 4 = X$
6. $6X = 40 - 4$
7. $25 - X = 15$
8. $\frac{100}{X} = 10$
9. $25 - X = \frac{100}{X}$
10. $50\%\ (X) = 12$ or $.5X = 12$
11. $X = 17 + 5$ or $X - 17 = 5$
12. $X - 12 = 2$
13. $X^2 = 2X$
14. $(X - 7) + 9$
15. $X + (X + 1) + (X + 2) = 3(X + 1)$

Handout

Translation Exercise 2

Variable assignment: Let X represent the unknown number in each sentence. Let Y represent the second number if necessary.

1. The difference between two numbers is 18.
2. One-half of a number is 10.
3. Six more than twice a number is 14.
4. The product of two numbers is 24.
5. The product of 4 and a number is equal to 21 plus that number.
6. Seventy-five percent of a number is 12.
7. Twenty-seven is 4 more than a number.
8. The quotient of 80 and 4 is 6 less than a number.
9. Three times a number plus 4 equals 5 times the number.
10. One-half of 10 times a number is 4 more than 3 dozen.

Handout

Answers to Translation Exercise 2

1. $X-Y = 18$
2. $\frac{1}{2}X = 10$
3. $2X + 6 = 14$
4. $XY = 24$
5. $4X = 21 + X$
6. $75\%\ (X) = 12$ or $.75X = 12$
7. $27 = 4 + X$
8. $\frac{80}{4} = X-6$
9. $3\ (X + 4) = 5X$
10. $\frac{1}{2}(10X) = 4 + 3\ (12)$

Lesson 10

Discovering Structure underlying Algorithmic Story Problems

Rationale

Understanding the structure underlying algorithmic problems helps students to make sense out of what they are doing. It provides them with a plan of attack for solving problems.

Objective

1. Each student will review and use structure to organize and solve three types of algorithmic problems: distance/rate/time, work, and mixture problems.

Materials and Resources

Handouts

"Summary of Designs for Story Problems"
"Auxiliary Skills: Problem Solving," from Lesson 6

Instruction and Activities

1. Thoroughly review with students the three designs on the handout "Summary of Designs for Story Problems." Relate *structure* to an attack plan for problem solving on the "Auxiliary Skills" handout.
2. Have students try to solve examples in each category by themselves before discussing them. Note that actual "solving" calculations have not been included on the handout, and that students should include the missing steps in their own solutions.

Evaluation

Have students make up similar story problems for the entire class to try.

Handout

Summary of Designs for Story Problems

Continue this list as you find more story problems to which you can relate each design or structure.

1. **Distance/Rate/Time Problems**

Chart

	Distance	Rate	Time
Vehicle 1			
Vehicle 2			

(Sometimes totals are necessary!)

Formula: $D = rt$ or $t = \frac{D}{r}$ or $r = \frac{D}{t}$

Notes

For "same" time, rate, or distance, use the same variable in each row.
For vehicle and wind or water current, state the vehicle speed *first*, then ± current speed.

Example

The current of a stream is 4 mph. A boat travels 6 miles upstream in the same time it takes to travel 12 miles downstream. What is the speed of the boat in still water?

Variable assignment: Let r represent the rate of the boat.
Let t represent the time in hours.

Chart

	Distance	Rate	Time
Up	6	$r-4$	t
Down	12	$r+4$	t

Solution example:

$D = rt$ or $t = \frac{D}{r}$ — 2 equations with 2 unknowns. Solve by addition or substitution methods.

$6 = (r - 4)t$ $\quad t = \frac{6}{r - 4}$ time up

$12 = (r + 4)t$ $\quad t = \frac{12}{r + 4}$ time down

time up = time down

$$\frac{6}{r - 4} = \frac{12}{r + 4}$$

$$\begin{aligned} 6(r + 4) &= 12(r - 4) \\ 6r + 24 &= 12r - 48 \\ 72 &= 6r \\ r &= 12 \end{aligned}$$

Answer: Speed of boat in still water is 12 mph.

2. Work problems

Main idea: Ratio of $\frac{1 \text{ task}}{X \text{ hours}}$

Chart

Rate person *A*	Rate person *B*	Rate *A*+*B* together

Formula: $\frac{1}{A} + \frac{1}{B} = \frac{1}{t}$

Example

R. B. builds an adobe wall of a certain size in 6 hours. It takes L. M. 8 hours to construct the same wall. Working together, how long will it take them to do the wall?

Variable assignment: Let t represent the number of hours it takes working together.

Chart

Rate of *A*	Rate of *B*	together
$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{t}$

Solution example:

$$\frac{1}{A} + \frac{1}{B} = \frac{1}{t}$$

$$\frac{1}{6} + \frac{1}{8} = \frac{1}{t}$$

$$8t + 6t = 48$$

$$14t = 48$$

$$t = 3\frac{3}{7}$$

Solve by clearing fractions or by combining like terms (LHS) and using the idea of proportion.

Answer: Working together it will take $3\frac{3}{7}$ hrs.

3. Mixture problems (Solutions, Tickets, Investments, or Coins)

Chart

	Item *A*	Item *B*	Total
Number of			
Value of			

Formula: number of item *A* + number of item *B* = total number
value of *A* + value of *B* = total value

Notes

If *d* represents the number of dimes then .10*d* represents the value of those dimes (\$). If *A* represents the number of liters of solution *A* and solution A is 35% alcohol, then .35*A* represents the amount of alcohol in solution *A*.

Example

I have 6 coins in dimes and nickels that total 50 cents. How many dimes and how many nickels do I have?

Variable assignment: Let *n* represent the number of nickels.
Let *d* represent the number of dimes.

Chart

	Nickels	Dimes	Total
Number of	*n*	*d*	6
Value of	.05*n*	.10*d*	.50

Solution example:

$n + d = 6$
$n = 6 - d$

2 equations with 2 unknowns. Solve by addition or substitution methods.

$.05n + .10d = .50$
$5n + 10d = 50$

Multiply by 100 to clear decimals.

using substitution

$$\begin{aligned} 5(6 - d) + 10d &= 50 \\ 30 - 5d + 10d &= 50 \\ 5d &= 20 \\ d &= 4 \\ n &= 6 - 4 \\ n &= 2 \end{aligned}$$

Answer: 2 nickels
4 dimes

Lesson 11

Reviewing the Origins and Structure of the System of Algebra

Rationale

The history of algebra is as fascinating as it is overwhelming. Much of the algebra performed today by developmental-level math students has taken centuries to develop. A struggling student might appreciate this fact. Having students explore the algebraic system is also invaluable.

Objectives

1. Students will be introduced to the origins of algebra.
2. Students will be introduced to the structure of a mathematical system.

Materials and Resources

Handouts

"The History of Algebra"

Instruction and Activities

1. Distribute "The History of Algebra" and instruct students to read the paragraphs.
2. Encourage students to explore algebra as a structural system of elements, operations, and rules. Discuss.

Evaluation

Ask students to identify the purpose of this lesson. See if they can relate this purpose to learning other new concepts.

Handout

The History of Algebra*

Traditionally, algebra has been defined to mean generalized arithmetic where letters represent numbers. For example, $3 + 3 + 3 + 3 = 4 \times 3$ is a special case of the more general algebraic statement that $Y + Y + Y + Y = 4 \times Y$. The name *algebra* comes from an Arabic word, *al-jabr*, which means "to restore."

A famous Moslem ruler, Harun al-Rashid, the caliph made famous in *Tales of Arabian Nights*, and his son, Al-Mamun, brought to their Baghdad court many famous Moslem scholars. One of these scholars was the mathematician Mohammed ibn-Musa al-Khowarizmi, who, about A.D. 800, wrote a text entitled *ihm al-jabr w' al-muqabal* (*al-muqabal* means simplification). The text included instructions for solving equations by adding terms to both sides of the equation, thus restoring equality. The abbreviated title of al-Khowarizmi's text, *Al-jabr*, became our word for equation solving and operations on letters standing for numbers—algebra.

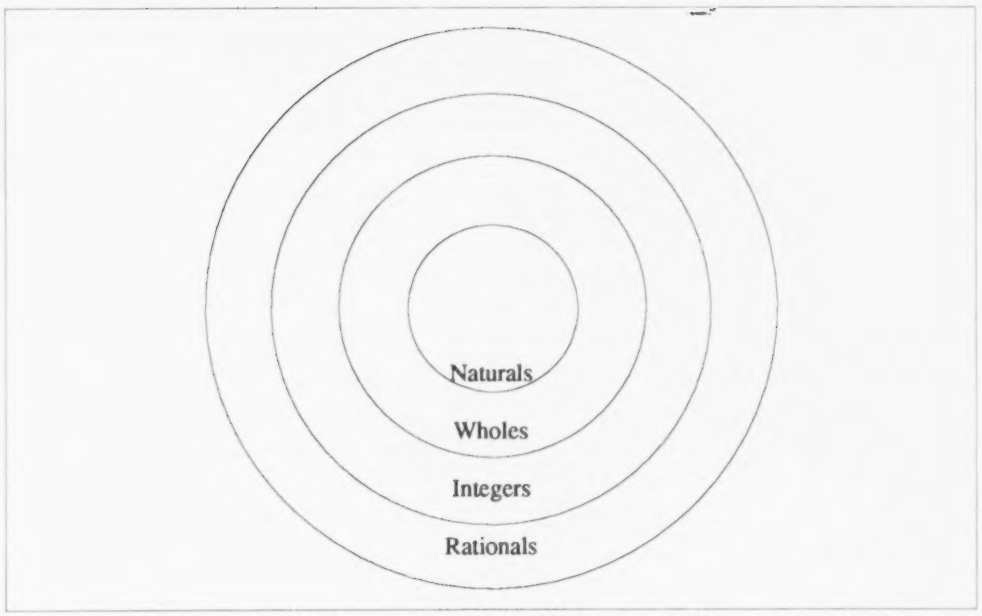
Al-Khowarizmi's algebra was brought to Western Europe through Moorish Spain in a Latin translation done by Robert of Chester (ca. A.D. 1140). Al-Khowarizmi's name may have sounded familiar to you, it was eventually translated as *algorithm*, which came to mean any series of steps used to solve a problem. Thus we speak of the *division algorithm* used to divide one number by another. Al-Khowarizmi is known as the father of algebra, just as Euclid is known as the father of geometry.

Components of a Mathematical System (Algebra)

1. Elements: "things" we work with
2. Operations: "what we do" with the elements
3. Rules: "how we do it"

Elements:	Real numbers are all the numbers represented by the number line, including naturals, whole numbers, integers, rationals, and irrationals.
Operations:	binary: $+, -, \times, /$; unary: $|\ |, -, \sqrt{}$, etc.
Rules:	Properties of real numbers: associative, commutative, distributive, identities, inverses Principles of exponents Order of operation Arithmetic (fractions, decimals) And so on

* From D. F. Wright and B. D. New, *Introductory algebra* (Boston: Prindle, Weber, and, Schmidt, 1978).



The Real Numbers

N = Naturals {1, 2, 3, ...}
W = Wholes {0, 1, 2, 3, ...}
J = Integers {..., -2, -1, 0,1, 2, 3, ...}
Q = Rationals {a/b a ∈ J, b ∈ J, b ≠ 0}
$\overline{Q}$ = Irrationals {π, √2, ...}
R = Reals Q U $\overline{Q}$ (all of the above)

Lesson 12

Preparing for and Taking Quantitative Exams

Rationale

A preexam routine helps students to organize their time and materials. Demonstrating active reviewing provides them with techniques to use. Students frequently miss points on exams because they do not know how to take exams.

Objectives

1. Students will develop an organized method of review for exams.
2. Students will demonstrate methods of active review.
3. Students will recognize factors that assist them in taking quantitative exams.

Materials and Resources

Index cards (five cards per student)

Handouts

"Preparing for a Comprehensive Final: Organizing Your Review"
"Weekly Schedule," from Lesson 1 (two or three copies per student)
"Review Organizer"
"Preparing for a Comprehensive Final: Studying Actively"
"Taking Quantitative Exams"

Instruction and Activities

1. Read the introduction to the handout "Preparing for a Comprehensive Final: Organizing Your Review."
   a. Distribute to each student two or three blank copies of the "Weekly Schedule" and a "Review Organizer."
   b. With students, go through steps 1–7 of the instructions for completing the "Review Organizer."
   c. Ask students to complete the "Review Organizer." As they review each topic, have them put a check in the completed column.
2. Go over the suggestions on the handout "Preparing for a Comprehensive Final: Studying Actively."
   a. Ask students to make five review cards.
   b. Have students write down three different ways that they actively study mathematics.
   c. Ask students to explain a concept to another student as a way of reviewing.
3. Review with students the handout "Taking Quantitative Exams."
   a. Ask students to identify which suggestions they need to adopt into their test-taking routine.
   b. Have each student write down a goal statement for the next exam, identifying the technique she or he will work on. Collect the papers.

Evaluation

1. At the end of a week, ask students to evaluate for themselves how well they are keeping to their exam review schedule. Ask them to write suggestions as to how they could improve.
2. Check students' index cards to ensure that they are done correctly. Ask students to list the ways they have studied actively.
3. Following the exam, have students rate themselves on how well they improved their exam-taking techniques. Also ask students to write down how they will improve their exam-taking techniques on the next exam.

Handout

Preparing for a Comprehensive Final: Organizing Your Review

Preparation for a comprehensive final really begins on the first day of class and continues through the semester. It involves keeping up with the daily assignments and asking questions when you do not understand something. It also involves weekly reviews of the material you have studied and the problems you have worked.

An intensive review should begin a few weeks before the exam. The following is a step-by-step procedure for organizing your intensive review. You will need your book, your notes, your homework assignments, summary cards, a calendar, a blank "Review Organizer" sheet, and several blank "Weekly Schedule" sheets.

1. Go through your book, your notes, your homework assignments, and summary cards, and record on the "Review Organizer" all the topics or major concepts you need to review for the exam. Include page numbers where appropriate or lecture dates.

2. Look at your calendar and count the number of study days you have left until the exam. Record the number on the bottom of the "Review Organizer."

3. Count the number of topics you need to review and record this number also.

4. Divide the number of topics by the number of study days. Doing so will tell you how many of these topics you need to review each day. Record this information at the bottom of the "Review Organizer."

5. Using the "Review Organizer," record under the column "Study Date" which day you are going to review each topic.

6. Complete a blank "Weekly Schedule" for each week remaining until the exam.

   a. First write in all your set commitments, such as class times, work, clubs, church, errands, and family activities.

   b. Write in mealtimes and some recreation time for yourself.

   c. Write in study times for your classes (one to three hours outside class for every hour in class).

   d. Looking at the times you have left each day, assign review times for your final.

7. Record the review times for each topic under the column "Time" on the "Review Organizer."

Handout

Review Organizer

List of Topics	Study Date	Time	Completed

Number of days left _____
Number of topics _____
Number of topics ÷ number of days = _____ topics per day

Handout

Preparing for a Comprehensive Final: Studying Actively

Once you have organized your review time, you need to organize what you are going to do when you sit down to study. If you really want to learn mathematics (or anything, for that matter), you must study and review actively. This means you must use your senses, say things out loud, write information down, draw charts, ask questions, summarize, and get involved. You learn best by doing, not by sitting back. Generally speaking, it takes more than reading about something to really understand it.

Here are some active ways of reviewing:

1. Make index review cards for each topic (like a "cheat" sheet).

   a. Write the topic on one side, and on the other side write steps to solve a problem, definitions, rules, and so forth.

   b. Test yourself by going through the cards and looking at the topics and then saying or writing down as much as you know about the topic without looking at the other side of the card.

2. Take the practice tests at the ends of your chapters. If you do not understand something, put a question mark by it or write it down on a piece of paper. Ask someone to clarify it for you.

3. Go over homework problems, particularly those you got wrong. Do not just look at what you have done; rework the problems.

4. Go over the old tests. Rework the problems to make sure you understand how you got your answers.

5. Explain to someone else how to do the problems.

6. Draw diagrams or charts to help yourself understand concepts.

7. Go through examples in the book step by step. Make sure that you understand how each answer is calculated.

8. Review your cards from the previous study sessions.

Handout

Taking Quantitative Exams

1. Get plenty of rest before the exam.
2. Eat a moderate amount so that you feel full but not sleepy.
3. Simplify your life as much as possible right before the exam. Do not take on added responsibilities, and avoid controversial topics with your family and friends. Treat yourself very carefully—pamper yourself—so that you can go to the exam feeling good.
4. Find a good spot to sit so that you will be free from distractions.
5. Practice some relaxation techniques while you are waiting for the exam.
6. When you receive the exam, read the directions carefully. Look through the entire test before you begin on any part. Notice the point value of the questions. Plan to spend more time on the questions that are worth more points.
7. Jot down formulas.
8. Answer first the questions that you know. Skip the ones you do not know, and go back to them later.
9. Show all your work and do not skip steps.
10. Draw diagrams to help yourself solve problems.
11. Check your answers when you are finished to ensure that they make sense. When you check your answers, *rework* the problems to see if you get the same answer.
12. Do not rush to finish when you see others finishing. Remember, there is no prize for being the first one to hand in the test. It is more important to pace yourself so that you will finish on time, not early.
13. Do not spend all your time on difficult problems that you cannot solve, unless you have finished all the other questions that you can solve.
14. If you do run out of time, try to set up the problem, even if you cannot finish it. You *may* get partial credit for your answer.
15. Proofread your answers to check for any errors or oversights.

Appendix A

Correspondence to University Support Staff and Instructors concerning the Math Learning Project

Introductory Letter for the Math Learning Project Course

CENTER FOR LEARNING ASSISTANCE

Box 5278/Las Cruces, New Mexico 88003
Telephone (505) 646-3136

WEEA Mathematics Learning Project

January 27, 1986

To:

From: Susan C. Brown, Project Director

I would like to introduce you to a new project on campus, the WEEA Mathematics Learning Project.

A proposal entitled "A Comprehensive Program to Assist Targeted University Students Cope with Math Anxiety" was funded by the Department of Education under the Women's Educational Equity Act. The goals of this one-year project are to provide the opportunity for selected women and minority students to examine the situations and attitudes that have created anxiety toward mathematics, to work with other students who have similar problems, to relearn specific mathematics concepts that present barriers to successful completion of higher mathematics courses, and to create an academic environment in which these students can successfully participate.

During spring of 1986, we will assess the needs of a target group of women and minority students who are struggling with mathematics. We hope to identify 30 to 40 students through recommendations from the academic support offices and advisement counselors on campus. These students will be interviewed. Twenty of these identified students will then participate in a special 3 credit course, fall semester 1986, which will provide assistance in mathematics through group instruction, group counseling, individual counseling, tutoring, and supervised study.

Upon completion of fall semester, the project will be evaluated and a manual will be written describing the components and the results. Student follow-up will be conducted.

We are looking forward to maintaining an open dialogue with your office throughout the project and welcome your recommendations in the following areas:

1. Your observations concerning mathematics learning that might benefit our project design and goals.
2. Students who could work as Research Assistants (Spring 1986, 5 hrs/wk; Fall 1986, 15 hrs/wk).
3. Students you think might benefit from extra academic support in mathematics fall semester 1986. Our course would be concurrent with other math/science courses the student might take.

Also working on the project will be Margaret Scott and Sandra Geiger. The project office is located in Jacobs 118: phone messages will be handled by the Center for Learning Assistance, 646-3136.

Needs Assessment Form

WEEA Mathematics Learning Project

A staff member will make an appointment at your convenience to discuss these items.

1. Your observations concerning academic support in mathematics learning and your suggestions to enhance our project.

2. List of students you would recommend for peer tutor positions.

3. List of students you recommend as project participants. Please include any related information on these students.

Student Interview Application for Student Needs Assessment

CENTER FOR LEARNING ASSISTANCE

Box 5278/Las Cruces, New Mexico 88003
Telephone (505) 646-3136

WEEA Mathematics Learning Project

February 20, 1986

Student Application for Mathematics Learning Project Interviews

We are conducting interviews on mathematics learning attitudes and situations as part of a federally funded project. These interviews will help us design a support course to be offered fall semester. Participants must be enrolled in Math 100, 102, or 115 and be experiencing difficulty or anxiety in learning mathematics. The interviews will take approximately one hour. We will pay you $5 for your participation.

All information from the interviews will be kept in the strictest confidence. If you are willing to participate please return the form below by March 7, 1986. Upon receipt of the form we will contact you to arrange an appointment.

****

Name _____ S. S. No. _____

Address _____

Phone _____ (best time to call) _____

Presently enrolled in Math _____

GPA _____ Circle class: freshman sophomore junior senior

Referred by: _____

Return to: Dr. Susan C. Brown, Director
Mathematics Learning Project, Box 5278

Appendix B

Tutor Information

Job Description

Student Assistant: WEEA Math Learning Project

A List of Recommended Job Qualifications

1. Student who can work both sessions as advertised: Spring 1986 (5 hours/week and possibly more) and Fall 1986 (15 hours/week).

   Possible summer partial employment.

2. Student with knowledge of Math 100, 102, 115 curriculum, who feels confident communicating this curriculum creatively to learners who are experiencing difficulty with mathematics.

3. Student who has had previous experience teaching or tutoring math and who enjoyed it.

4. Student who has empathy for others who struggle with understanding mathematical concepts.

5. Student who is sensitive to others and patient, and has good communication skills.

6. Student who believes in educational equity in mathematics for ethnic minorities and women.

7. Student who has worked previously with ethnic minorities or returning adult students.

8. Student who can work without supervision on a project.

Interviewing Techniques

Your first assignment will be to assist the staff in conducting interviews with students enrolled in Math 100, 102, and 115 who are experiencing difficulty learning mathematics. These students have volunteered to discuss these difficulties and, in some cases, might be very upset.

The goal of the interview will be to help the student identify his or her own concerns about learning mathematics, and possible reasons why he or she experiences difficulty. The interviewee will be paid $5 for her or his participation. As interviewers, we would like to emphasize the value we place on the interviewee's experiences, and how important this information is to designing the course we propose.

There are techniques you can use when interviewing to make the interviewee feel comfortable and safe. To communicate a positive attitude toward the interviewee, you can:

1. smile frequently
2. use eye contact*
3. maintain a closeness by leaning toward the interviewee
4. face the interviewee
5. maintain a relaxed position, arms open

We would ideally like the interviewee to do 80% of the talking, so we will use a substantial number of nondirective questions such as: "What do you think about ...?"

To encourage and show you support the interviewee, show interest and involvement in what she or he is saying through positive gesturing (like nodding your head), or verbal indication of understanding ("uh huh," or "I know what you mean," etc).

Try to avoid verbal cuing and imparting your own bias into a question.

Using specific interview questions, ROLE PLAY an interview.

* * *

Procedure

1. Introduce yourself. Make yourselves comfortable.
2. Statement on goal of interview.
3. Interview.
4. Statement of appreciation. Payment.
5. UNIV 112 registration.

*Might be awkward for American Indian students.

Tutoring Log

MATHEMATICS LEARNING PROJECT

Participant _____

UNIV 112 time _____

Day	Time	Comments	Tutor

Appendix C

Interview Questionnaire for Designing the Math Learning Project Course

Instructions to Interviewers

Name _____

Int ____

Math Learning Project Interviews
March 31, 1986-April 15, 1986

Introduction

1. Name of interviewee _____. Date _____. Time _____. Place _____.

2. Introductions: interviewee/interviewer. "Hi, I'm _____, project assistant with the Mathematics Learning Project, and I'll be conducting your interview today."

   Give the interviewee the demographic card to fill out. Make sure he/she includes his/her current math course (space after Question 13).

   Take the card when it is completed, read it over, and chitchat about the information on the card.

3. Statement on the goal of the interview: "The Math Learning Project is designed to assist women and minority students enrolled in Math 100, Math 102, or Math 115 in the learning of mathematics. Over the next two weeks, we are interviewing 40 students who are having difficulty in mathematics. These interviews will help the project staff design a course for fall semester 1986 that will provide assistance to students enrolled in the courses mentioned.

   "The goal of the interview will be to help *you* identify your own concerns regarding the learning of mathematics, possible reasons why you are experiencing difficulty, and most important, future avenues to success.

   "We would also like you to realize that your experience, ideas, and responses in these areas are shared by many people, and in this aspect, you are a spokesperson for the hundreds we cannot interview."

4. Description of interview: "The interview is divided into three parts—1. Past, 2. Present, and 3. Future—because studies have revealed that difficulty with mathematics does not happen overnight, but is a situation that is both created and developed throughout a person's life, often unconsciously.

   "The interview will consist of some short-answer questions where I will take notes, and some 'complete the sentence' or longer answer questions, where I will ask you to write down your ideas [give the interviewee her/his answer sheet]. We'll proceed through the questions in order, 1 through 17."

Questionnaire

Name _____

Interview Form Used by Interviewer

Instructions: Interviewee will write answers to questions marked **. First read the question, then let him/her write awhile. When she/he is done, ask her/him to read her/his answer aloud. Discuss. He/she may add more information. Interviewer may also take notes.

Interviewer takes notes on all other questions in space provided. Avoid making your own comment—too time consuming.

* * *

**Question 1. Complete the sentence: "An early experience I remember having to do with math was _____."

Question 2. What can you say about the attitudes toward mathematics that members in your family have?

a. Father

b. Mother

c. Brothers/Sisters

d. Spouse/Roommate

e. Children

Question 3. How did your high school friends feel about mathematics?

Question 4. How do your current friends feel about mathematics?

Name _____

**Question 5. Complete the sentence: "The hardest part or topic in math was/is _____, and the easiest was/is _____."

**Question 6. Complete the sentence: "I think I developed my present attitude toward mathematics when _____."

What is your present attitude?

Question 7. Describe a good math teacher. What should a good math teacher do?

Question 8. What is different about studying for math versus studying for other subjects?

Question 9. Regarding your own learning style, what techniques do you use that work when you study math?

**Question 10. Complete the sentence: "When I get stuck on a problem, or don't understand the lesson in math, I _____."

Name _____

Question 11. Do you ever get anxious about math? When? What happens? How do you get over it?

**Question 12. Describe how you successfully use math in your daily life outside the math classroom.

Other classes?

Job?

Elsewhere?

Question 13. Compare the math required above in Question 12 with the math used in your math class. Are they related?

**Question 14. Complete the sentence: "I want to have stronger math skills because _____."

What are your career or educational goals? How does math fit in here?

Name _____

Question 15. If you could design an ideal math class for yourself, what would it be like? Consider:

content

setting

population addressed

materials

Question 16. Would you be interested in enrolling in UNIV 112, Sec. 2 in Fall 1986?

Last
**Question 17. Are there any other comments you would like to make?

(At this time collect answer sheets from the interviewee and staple or attach to the larger interview packet. Make sure interviewee's name is on each page.)

Conclusion

Thank the interviewee for his/her participation in the interview. Give him/her a copy of the UNIV 112 announcement and go over it. If the interviewee is immediately interested in enrolling, have him/her fill out the bottom, tear off, and give to you. The student keeps the top.

Payment of $5 for the interview will be ready for pick-up at the Center for Learning Assistance (Hardman 210, 646-3136) by Friday, April 11, after 2:00 P.M. Suggest they call first.

Name _____

Math Learning Project Interviews

Questions to Be Written by the Interviewee: Use as much room as you need.

**Question 1. Complete the sentence: "An early experience I remember having to do with mathematics was _____."

**Question 5. Complete the sentence: "The hardest part or topic in math was/is _____, and the easiest was/is _____."

**Question 6. Complete the sentence: "I think I developed my present attitude toward mathematics when _____."

**Question 10. Complete the sentence: "When I get stuck on a problem, or don't understand the lesson in math, I _____."

**Question 12. Describe how you successfully use math in your daily life outside the math classroom.

Other classes?

Job?

Elsewhere?

**Question 14. Complete the sentence: "I want to have stronger math skills because _____."

**Question 17. Please record any other comments you would like to make.

Appendix D

Student Activities

Sample Journal Entries

I had to repeat Quiz in Chapter 5 today. I haven't had any real bad anxiety toward math until now. In this math class I've had a lot of motivation to do my studying. I think it's because I've been doing so well. Now that I've had to retake a quiz, my enthusiasm is not so high. I keep telling myself to stay positive and this will be OK. One quiz is nothing! I will give myself a little reward for not giving in to my negative thoughts. I've been here (MLC) since 8:30 A.M. and I'll stay until 3:30 P.M.

* * *

I started out trying to complete a daily diary of how I was feeling about math.

At the beginning I was really apprehensive about the course. Now I have finished it and have a lot more confidence in what I can do.

I've learned that with the next course I will start right from the beginning making notes and cards.

The class has helped me with a lot of tips that I know will be helpful next semester. Also knowing that I will have access to the study lab is a big help.

I started out with quite a deficit in math but I feel I'm catching up.

Even though I was completely frustrated at the word problems I can look back on it and see that it will not be as frustrating when I actually work them. The frustration that came through that morning had been building up for many years.

* * *

Just left the first session. It made an impression. I *hope* it helps. I noticed almost everyone in the program is overweight. Interesting. Overweight in my mind is a basic image problem. We're all dealing with an image problem, really, when the bottom line is drawn.

Next Thursday:

1. What is best—self-paced or lecture?
2. What is the simplest form of a fraction?
3. Why is $n^0 = 1$?
4. Distributive laws when factor $(1/2)a + (1/2)b$.
   I wrote $1/2\ (ab)$.
   The answer is $1/2\ (a + b)$.

* * *

17 Nov 86 - Study open book on Quiz 12 having lots of problems with algebra.
18 Nov 86 - After several hours and days of review with Algebra I seem to understand a bit more.
20 Nov 86 - Algebra the scale of balance. What you do on one side, you must do on the other.
21 Nov 86 - It's getting hard to remember the rules for each and every problem.
24 Nov 86 - Today I got a real good feeling about taking Quiz 12 tomorrow.
25 Nov 86 - Today I'm going to take Quiz 12 I feel with as much study & review as I've done I feel ready.

* * *

Tuesday very confident, but I froze on the intercepts on the exam. I truly surprised myself on the stated problem. I took your advice, Sandy—I gave everything a label and before I knew it all fell into place. (T.G. I can go on.) WHEW!

Student-Created Handout on Parentheses

Parentheses

by Gloria T. Lopez, Participant, Univ. 112-2

How to work a problem with parentheses:

$$[5\,(x + 3) - 2] - [6\,(y - 4) + 7]$$

Step 1: Take the equation within the first set of brackets.

$$[5\,(x + 3) - 2]$$

Multiply the 5 with the factors within the parentheses. This will remove the parentheses within the brackets.

$$[5x + 15 - 2]$$

Note: This does not remove the brackets.

Step 2: Take the equation within the second set of brackets.

$$[6\,(y - 4) + 7]$$

Multiply the 6 with the factors within the parentheses. This will remove the parentheses within the brackets.

$$[6y - 24 + 7]$$

Note: This does not remove the brackets.

Your equation should look as follows:

$$[5x + 15 - 2] - [6y - 24 + 7]$$

Step 3: To remove the brackets, take the first part of the equation, since there is no negative sign in front of the brackets. The brackets may be dropped and the equation written as is: $5x + 15 - 2$

Step 4: To remove the second set of brackets, note that there is a negative sign in front of the brackets. This negative sign should be multiplied against each term within the brackets: $-6 + 24 - 7$

Your equation should look as follows:

$$5x + 15 - 2 - 6y + 24 - 7$$

Step 5: Collect like terms. Since $5x$ and $-6y$ do not have any other like terms, they may be written as follows: $5x - 6y$

Step 6: Take the remaining positive-signed terms and add them as follows: +24 + 15 = 39

Step 7: Take the remaining negative-signed terms and add them as follows: -7 + - 2 = -9

Your equation should look as follows:

$$5x - 6y + 39 - 9$$

Step 8: Take the positive-signed number and negative signed number and subtract as follows: +39 – 9 = 30

Your final answer should look as follows:

$$5x - 6y + 30$$

Appendix E

Final Course Evaluation Questionnaire for the Math Learning Project Course

Math Learning Project Final Evaluation, Part I

Name _____

1. Complete the sentence: "The hardest part or topic in math was/is _____ and the easiest was/is _____."

2. How do you study math? How often do you study it?

3. Regarding your own learning style, what techniques do you use that work when you study math?

4. Complete the sentence: "When I get stuck on a problem, or don't understand the lesson in math, I _____."

5. Do you ever get anxious about math? When? What happens? How do you get over it?

6. What are your career or educational goals? How does math fit in here? Have your goals changed since you began the semester?

Math Learning Project Final Evaluation, Part II

	Not Helpful				Extremely Helpful
1. Learning about the situations and attitudes that contribute to math anxiety (Bill of Rights; information from research). Comments:	1	2	3	4	5
2. Learning about budgeting time. Comments:	1	2	3	4	5
3. Sharing your mathematics experiences with the class. Comments:	1	2	3	4	5
4. Writing the mathematics autobiography. Comments:	1	2	3	4	5
5. Keeping a mathematics diary/journal. Comments:	1	2	3	4	5
6. Opportunity to share one-on-one with instructor/tutor. Comments:	1	2	3	4	5
7. Learning how to study mathematics. Comments:	1	2	3	4	5
8. Completing the Mathematics Anxiety Rating Scale (both times). Comments:	1	2	3	4	5
9. Learning about Math Myths. Comments:	1	2	3	4	5
10. Problem solving using activities (logic puzzles, spatial puzzles). Comments:	1	2	3	4	5

	Not Helpful				Extremely Helpful
11. Problem solving using key words. Comments:	1	2	3	4	5
12. Learning about the language of mathematics (math vocabulary cards, dictionary, vocabulary worksheet). Comments:	1	2	3	4	5
13. Recognizing the importance of carefully reading mathematics. Comments:	1	2	3	4	5
14. Relaxation techniques. Comments:	1	2	3	4	5
15. Learning word problem designs (motion, work, coin-mixture, translations to math sentences). Comments:	1	2	3	4	5
16. Preparation for exams. Comments:	1	2	3	4	5
17. History of algebra. Comments:	1	2	3	4	5
18. Working in groups with other people who have similar problems with mathematics. Comments:	1	2	3	4	5
19. Other handouts (please specify). Comments:	1	2	3	4	5

Math Learning Project Final Evaluation, Part III

The following evaluation will assist us in refining this course. Please be as specific as you can in your comments. Be sure to evaluate the Math Learning Project course (UNIV 112), *not* your other math course.

1. Describe what your feelings about math were at the beginning of the Math Learning Project.
2. Describe what your feelings about math are at the end of the semester.
3. What are some specific math concepts that you have learned in the Tuesday morning sessions of the Math Learning Project that had been difficult for you in the past?
4. What did you like about the physical environment of Room 118, Jacobs Hall?
5. What would you change about the physical environment?
6. What did you like about the psychological environment of the Math Learning Project?
7. What would you change about the psychological environment?
8. What teaching techniques used were particularly helpful to you?
9. What teaching techniques were not helpful?
10. What tutoring techniques were particularly helpful?
11. What tutoring techniques were not helpful?
12. What specific study techniques did you learn that you would use in your next math class?
13. What is something of value you learned from writing the autobiography?
14. What is something of value you learned from keeping the diary/journal?
15. What is something of value you learned from keeping the notebook?
16. How many hours do you spend studying math outside of this class and outside of your other math class?
17. How many hours do you think you should spend?
18. What did you like best about the course?
19. What did you like least about the course?
20. What suggestions do you have for changing this course?
21. Would you recommend this class to a friend? Why or why not?

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